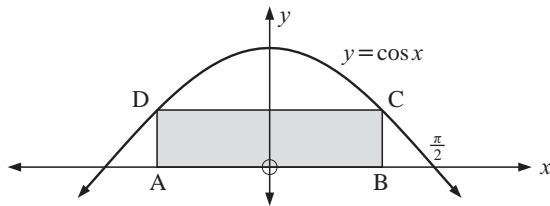


- b** When is the particle's:
i speed decreasing **ii** velocity decreasing?

- 13** Find the intervals where the function $f(x) = x^3 - 3x^2 - 9x + 5$ is:
a increasing **b** concave up.

14

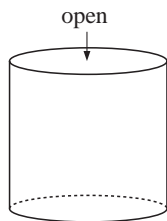


Rectangle ABCD is inscribed under one arch of $y = \cos x$. Find the coordinates of C such that ABCD has maximum area.

- 15** The kinetic energy of a moving object is given by $K = \frac{1}{2}mv^2$ where m is its mass in kg and v is its velocity in km s^{-1} . When a rocket is fired, the three quantities K , m and v vary. Suppose a rocket's kinetic energy is increasing at a constant rate of $50\,000 \text{ units s}^{-1}$ whilst its mass is decreasing at 10 kg s^{-1} . At what rate is its velocity changing when it has mass 4000 kg and velocity 8 km s^{-1} ?

- 16** Two roadrunners start at an intersection of two roads, facing 45° apart. One runs at 28 km h^{-1} and the other at 32 km h^{-1} . Find the rate at which the distance between them is changing after 15 minutes of running.

- 17** An open cylindrical bin is to be made from PVC plastic and is to have a capacity of 500 litres. Find the dimensions of the bin which minimise the amount of PVC plastic used.



18 Find $\int \frac{2x^2 - x - 3}{x^2} dx$.

19 Find the exact value of $\int_3^5 \frac{x}{x^2 - 8} dx$.

20 Find $\int \sin^2 3x dx$.

21 Find $\int \tan^2 2x dx$.

22 Find the exact value of k , if $\int_0^k \frac{x}{\sqrt{x^2 + 4}} dx = 1$, $k > 0$.

23 Use integration by parts to find $\int x \ln x dx$.

24 Use integration by parts to find $\int \arctan x dx$.

25 Find the exact value of $\int_0^4 \frac{1}{\sqrt{x+4}} dx$.

- 26** Find the total distance travelled by a particle in the first 5 seconds of motion if the particle is moving in a straight line and its velocity is given by $v = t^3 - 3t^2 e^{0.05t}$.

- 27** Find the exact area of the region enclosed by the graph of $y = 8x - x^2$ and the x -axis.

- 28** Find the exact value of the area of the region enclosed by the graphs of $y = x^2 - 3x$ and the line $y = x$.

- 29** Find the volume of the solid formed when the region enclosed by the graph of $y = x^2 - 2x$ and the x -axis is rotated about the x -axis. Give your answer as an exact value.

- 30** Find the volume of the solid formed when the region enclosed by the graph of $y = \ln x$, the x -axis, the y -axis and the line $y = \ln 3$ is rotated about the y -axis. Give your answer as an exact value.

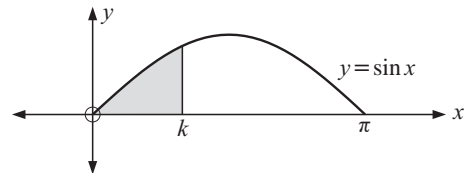
31 Solve for y : $y \frac{dy}{dx} = 2x - 3$, given that when $x = 0$, $y = 4$.

32 Solve for y : $\frac{dy}{dx} = 3x^2 y + y$, given that when $x = 0$, $y = 2$.

- 33** Integrate with respect to x :

a $x e^{-x^2}$ **b** 5^x **c** $(x^2 + 1)^3$

34



The shaded region has area 0.42 units^2 . Find k .

35 Find $\int_0^3 |x - 1| dx$.

36 By considering $\frac{d}{dx}(x^2 \ln x)$, find $\int x \ln x dx$.

37 Show that $\int \sec x dx = \ln |\sec x + \tan x| + c$.

- 38** Find:

a $\int \sec^3 x dx$ **b** $\int x \sqrt{3x - 4} dx$

39 a Write $\frac{x+2}{(x-1)(x+3)}$ in the form $\frac{A}{x-1} + \frac{B}{x+3}$.

b Hence find $\int \frac{x+2}{(x-1)(x+3)} dx$.

EXAM PRACTICE SET 1 NO CALCULATORS

- 1 a** The sum of the first eight terms of an arithmetic sequence is -4 whilst the sum of the next eight terms is 188. Find an expression for u_n , the n th term of the sequence.

- b** x and y are integers such that 9, x , and y are consecutive terms of a geometric sequence and x , y and 2 are consecutive terms of an arithmetic sequence. Find the values of x and y .

- 2** The coordinates of X and Y are $(3\mu, 2, 1)$ and $(\mu, 1 - 3\mu, 2\mu - 1)$ respectively, where μ is a constant. If O is the origin, find all values of μ for which \vec{OX} is perpendicular to \vec{OY} .

3 Let $f(x) = x^2 + 4x$ for $-\infty < x \leq -2$ and $g(x) = \sqrt{3 - 2x}$.

Find: **a** $f^{-1}(x)$ **b** $(g \circ f)(-3)$.

- 4** If the polynomial $x^n + ax^2 - 6$ leaves a remainder of -3 when divided by $(x - 1)$ and a remainder of -15 when divided by $(x + 3)$, find the values of a and n .

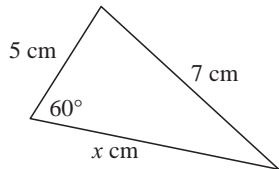
- 5** In triangle ABC, $\widehat{ABC} = 2\theta$, $\widehat{ACB} = \theta$, $AB = 3 \text{ cm}$ and $AC = 5 \text{ cm}$. Find the exact value of the area of the triangle.

6 Find the exact values of x if $-\pi \leq x \leq \pi$ and $\sin^2 x + \cos x = \frac{5}{4}$.

7 Find the exact value of x if $2^{2x} + 2^{x+1} - 15 = 0$.

- 8 When a cubic polynomial $p(x)$ is divided by $x(2x - 3)$, the remainder is $ax + b$, where a and b are real.
- If the quotient is the same as the remainder, write down an expression for $p(x)$.
 - Prove that $(2x - 1)$ and $(x - 1)$ are both factors of $p(x)$.
 - Find the equation of $p(x)$ given that it has y -intercept $(0, 7)$ and passes through the point $(2, 39)$.
- 9 Each of two circles with the same radius passes through the centre of the other. Find the area common to both circles if the radii are 6 cm, giving your answer as an exact value.

- 10 Find the exact value of x in the diagram alongside.



- 11 A discrete random variable X has the probability distribution:

x	0	1	2	3	4
$P(X = x)$	$\frac{1}{6}$	$2k$	$\frac{1}{5}k$	$\frac{1}{3}$	$\frac{2}{5}k$

- Find the exact value of k .
 - Calculate $P(0 < X < 4)$.
- 12 Find the exact values of the coordinates of the point of intersection of the normal at the point where $x = \frac{\pi}{6}$, to the graph of $y = \sin x$, and the x -axis.
- 13 The graph of the function $f : x \mapsto ax^3 + bx^2 + cx + d$ has a maximum turning point at $(0, 1)$ and a minimum turning point at $(-2, -2)$. Find a, b, c and d .
- 14 $[AB]$ is a diameter of a circle of radius 5 cm and P is a point on the circle. P moves at a constant rate in a clockwise direction on the circle and completes one rotation every 10π seconds. Find, as an exact value, the rate of change in the area of triangle ABP when $\widehat{PAB} = \frac{\pi}{3}$ and P is moving towards B .

- 15 Find the exact value of $\int_0^{\frac{\pi}{6}} (\sin 3x \cos 2x + \cos 3x \sin 2x) dx$

- 16 The region enclosed by $y = \arccos x$, $y = \frac{\pi}{3}$, $y = \frac{\pi}{6}$ and the y -axis is rotated about the y -axis. Find the exact value of the volume of the solid generated.

- 17 Use the substitution $x = \sin u$ to find the exact value of $\int_0^{\frac{1}{2}} \sqrt{1-x^2} dx$.

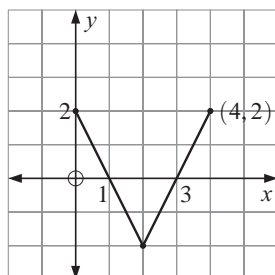
- 18 Find the exact value of p if $p > 0$ and $\int_0^p x^3 + x dx = \frac{15}{4}$.

- 19 The graph of $f(x)$ is given.

- Draw the graph of $|f(x)|$, on the same set of axes as shown.

- Find the y -intercept of $\frac{1}{f(x)}$.

- Draw the graph of $y = \frac{1}{f(x)}$ on the same set of axes as shown.



- 20 Consider the two lines with equations:

$$L_1: x = 3\lambda + 4, y = \lambda + 4, z = 2\lambda - 1,$$

$$L_2: x + 1 = \frac{y - k}{2} = \frac{z - 1}{-2}.$$

- Find the value of k for which these lines are coplanar, and hence determine the point of intersection of the two lines.
- Find a vector \mathbf{p} which is perpendicular to both L_1 and L_2 .
- Show that an equation of the plane which is normal to vector \mathbf{p} and contains L_1 is $-6x + 8y + 5z = 3$.

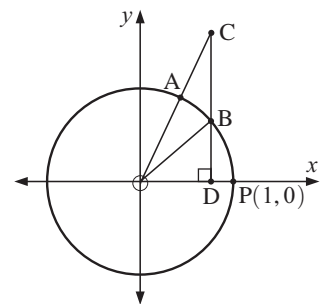
- 21 The function f is defined by: $f(x) = \frac{k \ln x}{x}$, $k > 0$, $x > 0$.

- Find the exact coordinates of the stationary point on the graph of $y = f(x)$.
- Find the exact coordinates of the point of inflection on the graph of $y = f(x)$.
- Find k such that $\int_1^{e^2} \frac{k \ln x}{x} dx = 10$.
- Use integration by parts to show that $\int \frac{(\ln x)^2}{x^2} dx = -\frac{1}{x}(\ln x)^2 - \frac{2}{x} \ln x - \frac{2}{x} + c$
- Hence, find the exact value of the volume of the solid formed by rotating the region enclosed by the x -axis, $y = f(x)$, where $k = 1$, and the line $x = e^2$ about the x -axis.

- 22 On the unit circle shown, $\widehat{AOP} = \alpha$ and $\widehat{BOP} = \beta$.

$[OA]$ is produced to meet the line through B parallel to the y -axis at C .

$[CB]$ is produced to meet the x -axis at D .



- Write down the lengths of $[OD]$ and $[BD]$.
- Show that $OC = \frac{\cos \beta}{\cos \alpha}$.
- Show that $BC = \cos \beta \tan \alpha - \sin \beta$
- Use the sine rule in triangle OBC to show that $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$.
- Hence find the exact value of $\sin 15^\circ$.

- 23 Find the exact value of $\tan x$ if $2 \sin(x + \frac{\pi}{6}) = \sin x$.

- 24 Find the exact value of the x -coordinate of the stationary point of the function $f : x \mapsto e^{x \ln x}$, $x > 0$.

- 25 Find the coordinates of the stationary points on the curve $y = \sin x(1 + \cos x)$ for $0 \leq x \leq 2\pi$.

- 26 The radius of the base of a right circular cylinder is r cm and its height is $2r$ cm. Find the rate at which the surface area of the cylinder is increasing when the radius is 5 cm and the volume is increasing at $5\pi \text{ cm}^3 \text{ s}^{-1}$.

EXAM PRACTICE SET 2 CALCULATORS

- 1 A geometric series with common ratio r , where $\frac{1}{2} < r < 1$ has second term 6 and a sum of 49. Find:

- r
- u_n , the n th term of the series.

- 2 Eight different books are to be placed in a row on a bookshelf. Three of the books are mathematics books. How many arrangements are possible if:

- there are no restrictions

- b** the mathematics books are always placed together
c there has to be a mathematics book at each end of the shelf?

3 Given the two sets of equations

$$\begin{cases} y_1 = x_1 + x_2 - x_3 \\ y_2 = 2x_1 - x_2 + 5x_3 \\ y_3 = 3x_1 - 2x_2 + x_3 \end{cases} \quad \text{and} \quad \begin{cases} x_1 = 2z_1 + 3z_2 - z_3 \\ x_2 = 4z_1 - 2z_2 + z_3 \\ x_3 = z_1 - 6z_2 + 7z_3 \end{cases}$$

use matrix methods to obtain three equations that express y_1 , y_2 , and y_3 directly in terms of z_1 , z_2 and z_3 .

- 4** A sector cuts off an angle of 53° at the centre of a circle of radius 7 cm. Find:
a the perimeter **b** the area of the sector.
- 5** Find the constant term in the expansion of $\left(2x^2 + \frac{1}{x}\right)^9$.
- 6** Let f and g be two functions such that $g(x) = 3x - 2$ and $(f \circ g)(x) = x + 2$. Find $f(9x - 8)$.
- 7** Find the points of intersection of the graphs of $x^2y = 4 + x$ and $y = e^x - 3x + 1$, where $-5 \leq x \leq 5$.
- 8** For what values of x in the interval $[0, 2\pi]$ is $\arccos(\sin 3x + \cos 2x)$ defined?
- 9** A receptionist walks to work every day.
 If it is not raining, the probability that he is late is $\frac{1}{6}$.
 If it is raining, the probability that he is late is $\frac{3}{5}$.
 The probability that it rains on a particular day is $\frac{1}{5}$.
 On one particular day the receptionist is late.
 Find the probability that it was raining on that day.
- 10** X is a binomial random variable, where the number of trials is 7 and the probability of success of each trial is p . Find the possible values of p if $P(X = 4) = 0.25$.
- 11** Simplify $\sin(\arcsin a + \arcsin b)$, giving your answer in terms of a and b .
- 12** **a** Find the inverse of the matrix $\mathbf{A} = \begin{pmatrix} -1 & k \\ k & 1 \end{pmatrix}$ where $k \in \mathbb{R}$.
b Hence, or otherwise, solve the simultaneous equations:
$$\begin{cases} -x + ky = 2k \\ kx + y = 1 - k^2 \end{cases}$$
- 13** Find a unit vector parallel to $\lambda\mathbf{i} + \mathbf{j} - \lambda\mathbf{k}$ and perpendicular to $3\mathbf{i} - 4\mathbf{j} + \mathbf{k}$.
- 14** In a school, $\frac{2}{7}$ of the students travel to school by bicycle. Seven students are chosen at random. Find the probability that exactly 4 of them travel to school by bicycle.
- 15** A box contains 1000 fish with 3.7% not suitable for sale. If 20 fish are chosen at random from the box, find the probability that:
a all of them are suitable for sale
b exactly one of them is not suitable for sale.
- 16** The coefficient of x^4 in the expansion of $(ax + 3)^5$ is equal to the coefficient of x^5 in the expansion of $(ax + 3)^7$. Find a .
- 17** Consider the functions $f(x) = \frac{1}{x+5}$, $x \neq -5$ and $g(x) = 3x$.
a Calculate $(g \circ f)(4)$ as a rational number.
b Find $g^{-1}(x)$. **c** Find the domain of g^{-1} .
- 18** Consider $\mathbf{A} = \begin{pmatrix} 1-k & -2 & -1 \\ 1 & -1 & -2 \\ 1 & k & -1 \end{pmatrix}$, $k \in \mathbb{R}$.

a For what values of k is the matrix non-singular?

b Find \mathbf{A}^{-1} if $k = 0$.

c Hence find the point of intersection of the planes: $x - 2y - z = 2$
 $x - y - 2z = -1$
 $x - z = 2$

d Show that the point of intersection of the planes lies on the line L given by: $\frac{x+3}{2} = y+4 = \frac{z-7}{-1}$.

e Find the angle between L and the plane $x - 2y - z = 2$.

19 A police radar gun measuring the speeds of cars on the road is known to produce errors at random that are distributed normally with mean 0 and standard deviation σ .

We say the error E is distributed normally as

$$E \sim N(\mu = 0, \text{variance} = \sigma^2).$$

The police will not prosecute without being 99% confident that a car is travelling at greater than 60 km h^{-1} in a 60 km h^{-1} zone. Suppose they are 99% sure that a car is travelling at greater than 60 km h^{-1} when they record a speed of more than 65 km h^{-1} on the radar gun.

- a** Explain why the distribution of readings for a car travelling at 60 km h^{-1} is given by the random variable $X = 60 + E$.
b Find the standard deviation σ of the distribution of errors given by the radar gun.

20 If $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, show that $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c}$.

21 The diameters of discs produced by a machine are normally distributed with a mean of 73 mm and standard deviation of 1.1 mm. Find the probability of the machine producing a disc with a diameter larger than 75 mm.

22 Independent events A and B are such that $P(A) = 0.35$ and $P(A \cup B) = 0.75$. Find $P(B)$.

23 A couple is told that the probability that they will have blonde haired children is $\frac{1}{7}$. The couple would like to have 5 children.

- a** What is the expected number of blonde children?
b What is the probability that 3 of the children will have blonde hair?
c What is the probability that more than 3 children will have blonde hair?

24 The probability that a salesperson leaves her mobile phone in a shop is $\frac{1}{6}$. After visiting two shops in succession, the salesperson discovers her mobile phone is missing. What is the probability that she left the mobile phone in the first shop?

25 **a** Find the exact value of the area enclosed by the curve

$$y = \frac{1}{\sqrt{4-x^2}}, \text{ the } y\text{-axis, the } x\text{-axis and the line } x = 1.$$

b The region in **a** is rotated about the x -axis. Find the volume of the solid generated.

26 When a biased die is rolled the numbers from 1 to 6 appear according to the following probability distribution.

x	1	2	3	4	5	6
$P(X = x)$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{3}{14}$	$\frac{1}{14}$	$\frac{1}{7}$	y

- a** Find the value of y .
b If X is the random variable which takes on values 1 to 6 of the above table, find the exact values of $E(X)$ and $\text{Var}(X)$.
c Write a brief interpretation of the values found in part **b**.