39 At time t seconds a particle is moving in a straight line with acceleration $4\cos t$ m s^{-2}. At $t = 0$ its velocity is 2 m s^{-1}.

a Find the velocity at $t = 4$ seconds.

b Find the total distance travelled in the interval $t = 0$ to $t = 5$ seconds. Give your answers to two decimal places.

40 Let $f(x) = 2\cos x - \sin x + 2x\sin x$, $0 \leqslant x \leqslant 2$.

a Show that $f'(x) = (2x - 1)\cos x$.

b Find the minimum value of $f(x)$ for $0 \leqslant x \leqslant 2$.

41 Find a given that $0 < a < 2\pi$ and $\int_a^{a+2} \sin x\, dx = 0.3$.

EXAM PRACTICE SET 1 NO CALCULATORS

1 **a** Differentiate with respect to x:

i $y = 3e^{x^2} + 3(2 - x)$ **ii** $y = \ln(2x + 5)$

b Find: **i** $\int \sin(\pi - x)\, dx$ **ii** $\int x(1 + x)\, dx$

2 A bag contains 4 green, 6 blue, and 2 yellow balls. Two balls are drawn at random from the bag without replacement. What is the probability that they are of different colours?

3 Given that $\log_7 A = x$, express each of the following in terms of x:

a $\log_7 A^3$ **b** $\log_7 \dfrac{1}{\sqrt{A}}$ **c** $\log_{49} A$

4 **a** Line L has the vector equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

Which of the following are also vector equations for the same line?

A $\mathbf{r} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ **B** $\mathbf{r} = \begin{pmatrix} -7 \\ 4 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

C $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + t\begin{pmatrix} -4 \\ -1 \end{pmatrix}$ **D** $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 12 \\ -3 \end{pmatrix}$

b A rocket is travelling with a speed of 800 km min^{-1} in the direction $\begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$.

At the time $t = 0$, the rocket was launched from the point P(2, 5, 0.05). Find the position vector $\mathbf{r}$ of the rocket at time t.

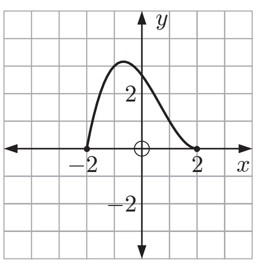
5 Consider the function $f(x) = \dfrac{4}{\sqrt{4 - x^2}}$, $-2 < x < 2$.

a Write down the equation of each vertical asymptote.

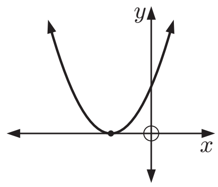
b Find the minimum value of the function f on the given domain.

c Sketch the graph of $y = f(x)$.

6 Copy the graph of $y = f(x)$ alongside.
On the same diagram, draw the graph of $y = 2\,f(x - 1) - 3$.



7 The graph of the function $y = ax^2 + bx + c$ is shown alongside.
Complete the table to show whether each expression is positive, zero, or negative.

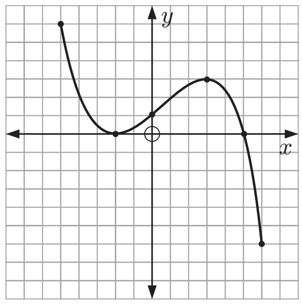


expression	*positive*	*negative*	*zero*
a			
b			
c			
$b^2 - 4ac$			

8 Given $f : x \mapsto x + \dfrac{1}{x}$ and $g : x \mapsto e^{-x}$, find:

a $g^{-1}(x)$ **b** $(f \circ g)(x)$

9 The diagram alongside shows the graph of $y = f(x)$.
On the same set of axes sketch the graph of $y = f'(x)$.



10 **a** The equation $kx^2 - 7x + 4 = 0$ has exactly one solution. Find the value of k.

b For what values of n will the equation $3\sin(2x - \pi) = n$ have no solutions?

c Find the gradient of the tangent to the curve $y = 2\cos(4x + \pi)$ at the point $(\frac{\pi}{8}, 0)$.

11 Given $\mathbf{a} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$, find:

a $|\,\mathbf{a}\,|$ **b** $2\mathbf{a} - 3\mathbf{b}$ **c** $\mathbf{a} \bullet \mathbf{b}$

d the vector equation of the line parallel to $\mathbf{b}$ that passes through $(1, 5)$.

12 **a** The quadratic equation $2x^2 - 3mx + 8 = 0$, $m > 0$ has exactly one solution for x. Find the value of m.

b The function h is defined by $h : x \mapsto \sqrt{4x + 9}$, $x \geqslant -2\frac{1}{4}$. Evaluate $h^{-1}(10)$.

13 In a survey, 80 students were asked "Do you prefer computer games or playing sport?". The results are partially displayed in the following table.

	Girls	*Boys*	*Total*
Computer Games	10	12	
Sport			
Total		36	

By completing the table, or otherwise, find the probability that a student selected at random:

a prefers to play sport

b prefers to play sport, given that the student is a girl.

14 Complete the following expansion
$(ax - 3)^4 = a^4x^4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + 81$

15 **a** Suppose $\mathbf{A}$ is the 3×2 matrix $\begin{pmatrix} 2 & 4 \\ 3 & 2 \\ -1 & 2 \end{pmatrix}$ and $\mathbf{AB} = \mathbf{C}$.

i State the possible orders of matrices $\mathbf{B}$ and $\mathbf{C}$.

ii Find $\mathbf{C}$, given that $\mathbf{B} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

b Expand $(\mathbf{A} + 2\mathbf{I})(\mathbf{A} - 3\mathbf{I})$ where $\mathbf{A}$ is a square matrix and $\mathbf{I}$ is the identity matrix.
Rewrite the equation $(\mathbf{A} + 2\mathbf{I})(\mathbf{A} - 3\mathbf{I}) = \mathbf{O}$ in the form $\mathbf{AB} = 6\mathbf{I}$. Hence write $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$ and $\mathbf{I}$.

16 **a** A water tank is to have a rectangular cross-section with dimensions $2x$ m by x m.

i If the volume of the tank must be 9 m^3, find an expression for the height y in terms of x.

ii Assuming the tank is closed at the top, show that the total surface area A of the tank is given by $A = 4x^2 + \dfrac{27}{x}$ m^2.

iii Find the dimensions of the tank for which the surface area is minimised.

b Let $f(x) = 1 - \dfrac{1}{1+x^2}$.

i Find the position and nature of the stationary point of the graph of $y = f(x)$.

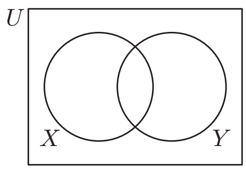
ii Find the position of any points of inflection.

iii By using a sign diagram or otherwise, state the values of x for which $f(x) \geqslant 0$.

iv Using all the information obtained above, sketch the graph of $y = f(x)$.

17 **a** At a book sale Josie bought five books by Jane Austen and two books by Catherine Cookson. She randomly selects two of the books to take with her on a holiday. Calculate the probability that both books are by the same author.

b This Venn diagram shows a sample space U and events X and Y.
$n(U) = 100$, $n(X) = 60$, $n(Y) = 70$, and $n(X \cap Y) = 42$.



i Find:
(1) $n(X \cup Y)$ **(2)** $\text{P}(X')$ **(3)** $\text{P}(Y \mid X')$

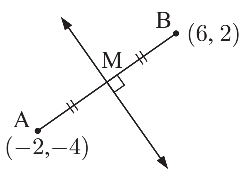
ii Explain why events X and Y are independent.

18 **a** The graph of $y = 10 - 3x$ intersects the x-axis at P and the y-axis at Q.

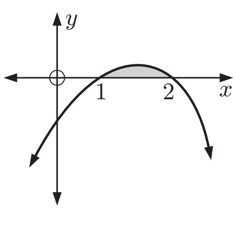
i Find the coordinates of P and Q.

ii Find the area of ΔOPQ where O is the origin.

b Find the equation of the line perpendicular to [AB] which passes through M, the midpoint of [AB].



c The sketch shows part of the graph of $y = x + \dfrac{2}{x-3}$.
Find the area of the shaded region from $x = 1$ to $x = 2$. Give your answer in the form $a + b(\ln 2)$ where a and b are constants.



19 **a** Given $\mathbf{v} = 3\mathbf{i} - 7\mathbf{j}$ and $\mathbf{w} = 5\mathbf{i} + 2\mathbf{j}$, find:

i $\mathbf{v} + 2\mathbf{w}$ **ii** a unit vector perpendicular to $\mathbf{v}$

iii $(\mathbf{v} + \mathbf{w}) \bullet (\mathbf{v} - \mathbf{w})$ **iv** $|3\mathbf{w}|$

b **i** Find vector equations for the following lines:

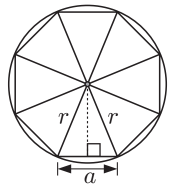
L_1 : passing through $(4, -11, 5)$ in the direction $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

L_2 : passing through $(10, -4, -2)$ and $(-2, 2, 16)$.

ii Find the coordinates of the point P where lines L_1 and L_2 intersect.

iii Find the cosine of the acute angle between the lines L_1 and L_2.

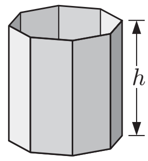
20 A circle of radius r encloses a regular octagon as shown.



a Show that a and r are related by $a = 2r\sin\left(\frac{\pi}{8}\right)$.

b Show that the area of the octagon is $8r^2 \sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$.

c



Octagonal waste paper bins are to be made of fixed volume V. The radius r (shown in the first diagram) and height h are to be determined.

i Show that the area of material needed for the base and sides is given by $A = 8r\left(2h + r\cos\left(\frac{\pi}{8}\right)\right)\sin\left(\frac{\pi}{8}\right)$ and that the volume $V = 8r^2 h \sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$

ii Show that $A = \dfrac{2V}{r\cos\left(\frac{\pi}{8}\right)} + 8r^2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$.

iii Show that A is a minimum when
$$r = \left(\frac{V}{8\sin\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)}\right)^{\frac{1}{3}}.$$

EXAM PRACTICE SET 2 CALCULATORS

1 In an arithmetic sequence the first term is 8 and the thirtieth term is 124.

a Find the 12th term.

b Find the sum of the first twelve terms.

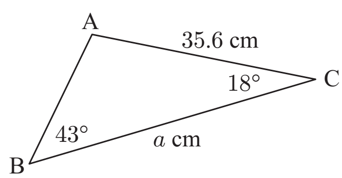
2 Given $\mathbf{A} = \begin{pmatrix} 2 & 7 & 6 \\ -1 & 3 & 2 \\ 4 & -2 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -3 & 1 & 2 \\ 2 & 1 & -5 \\ -2 & 4 & 3 \end{pmatrix}$ find:

a $\mathbf{A} - \mathbf{B}$ **b** $\mathbf{AB}$ **c** $|\mathbf{A}|$ **d** $\mathbf{B}^{-1}$

3 Find:

a a

b the area of ΔABC



4 Find the coefficient of the x^6 term in the expansion of $(2x+5)^{10}$.

5 **a** Solve $\begin{pmatrix} 1 & -3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$.

b **i** Find the possible values of p such that $\begin{pmatrix} 1 & p-2 \\ 2p & p^2 \end{pmatrix}$ has an inverse.

ii State this inverse in terms of p.

6 The heights of trees in a plantation are normally distributed with mean 3.18 metres and standard deviation 19.5 cm.

a Find the probability that a randomly selected tree will be:

i taller than 3 metres

ii between 2.8 and 3.3 metres tall.

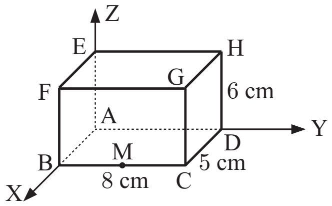
b What height must a tree be to be in the tallest 20%?

7 **a** Given $f(x) = 3\sin(2x+1)$, find:

i $f'(x)$ **ii** $\int f(x)\,dx$

b Find $\int e^x(1-e^{2x})\,dx$

8 The diagram shows a rectangular prism in which M is the midpoint of [BC]. Suppose A is at (0, 0, 0) and B is at (5, 0, 0).



a State the coordinates of E, M and G.

b Find:

i $\overrightarrow{ME}$ **ii** $\overrightarrow{MG}$

c Use a vector method to find the measure of $\text{E}\widehat{\text{M}}\text{G}$.

9 Consider $f(x) = \dfrac{1}{x^2-4}$, $-3 \leqslant x \leqslant 3$.

a Sketch a graph of $y = f(x)$.

b Write down the equation of each vertical asymptote.

c State the range of $f(x)$ on the given domain.

10 **a** Sketch the graphs of $y = \ln x$ and $y = 2 - x$ for $-1 \leqslant x \leqslant 4$.

b The equation $\ln x = 2 - x$ has a solution between -1 and 4. Find this solution.

11 **a** Factorise the expression $3\cos^2\theta - 10\cos\theta + 3$.

b Solve the equation $3\cos^2\theta - 10\cos\theta + 3 = 0$ for $0^o \leqslant \theta \leqslant 360^o$, giving your answers correct to the nearest degree.

12 A sum of 12 000 euros is invested at a compound interest rate of 7.2% per annum.

a Write down an expression for the value of the investment after t complete years.

b What will be the value of the investment at the end of 4 years?

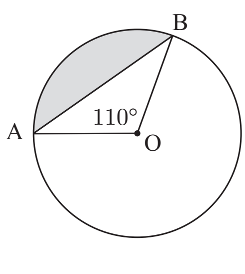
c How long will it take before the value of the investment is 25 000 euros?

13 Consider the points A$(-2, 0, 3)$, B$(1, -3, 5)$ and C$(7, 2, -1)$. Find:

a $|\overrightarrow{AC}|$ **b** a unit vector parallel to $\overrightarrow{AB}$

c the measure of $\text{C}\widehat{\text{A}}\text{B}$.

14 O is the centre of a circle that has a radius of 16 cm. Chord [AB] subtends an angle of 110^o at the centre of the circle.



a Find the length of [AB].

b Find the shaded area.

15 Let $f(x) = 4 - \sin 2x + 2\cos\left(\frac{x}{2}\right)$, where x is in radians. For the domain $0 \leqslant x \leqslant 4$:

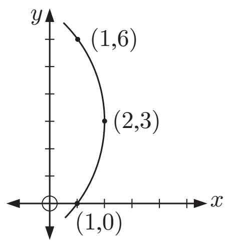
a sketch the curve of $y = f(x)$

b solve $f(x) = 5$

c find the minimum value of $f(x)$.

16 **a** If $\det\mathbf{A} = 0$ where $\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 1 & k & 2 \\ 2 & -1 & 1 \end{pmatrix}$, find k.

b A circular railway line is to pass through the three points shown in the diagram.

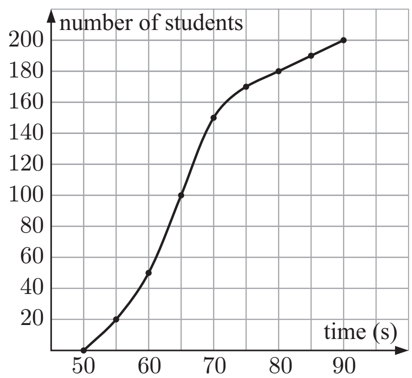


i By substituting each point into the equation of the circle $x^2 + y^2 + px + qy + s = 0$, show that p, q and s satisfy the system of equations:

$$\begin{cases} p + s = -1 \\ 2p + 3q + s = -13 \\ p + 6q + s = -37. \end{cases}$$

ii Find the values of p, q and s.

17 **a** The times taken by 200 students to complete a 400 metre run were recorded. The cumulative frequency graph for the times is given below.



i How many students took less than 60 seconds to complete the 400 metre run?

ii Find the median and interquartile range for the times taken by the 200 students.

iii Find the 90th percentile of the times taken by the 200 students.

b An agent for a football club contacts nine large corporations seeking sponsorships of \$10 000 each. The agent's record suggests the probability that she will obtain sponsorship from any given company is 0.3.

i What is the random variable in this case?

ii List the possible outcomes that could occur.

iii Find the probability that the agent gets:

(1) exactly 4 sponsorships

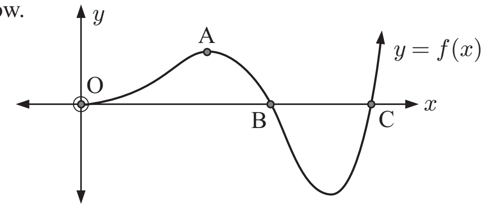
(2) at least 3 sponsorships.

iv The agent earns a commission of 2.5% per sponsorship plus a bonus of 1% for each sponsorship after the first. Her total costs are \$150. Calculate the probability that she makes a profit of \$1150 or greater.

v Now suppose the agent approaches 40 corporations instead of nine. Her costs increase to \$300. Calculate the probability that she makes a profit less than \$6600.

18 **a** If $f(x) = e^{3x} \sin 2x$, show that $f'(x) = 0$ when $\tan 2x = -\frac{2}{3}$.

b The graph of $f(x) = e^{3x} \sin 2x$ for $x \geqslant 0$ is shown below.



The graph intersects the x-axis at the points B and C. The point A is a local maximum.

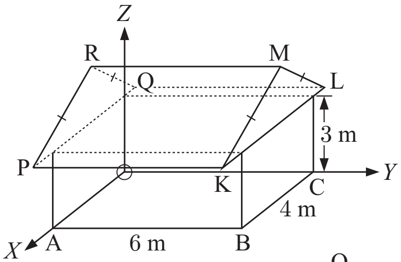
i Find the x-coordinates of B and C.

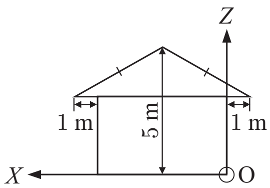
ii Find the x-coordinate of A.

c **i** If $g(x) = \frac{1}{13}(3e^{3x} \sin 2x - 2e^{3x} \cos 2x)$, show that $g'(x) = e^{3x} \sin 2x$.

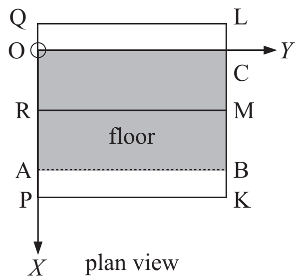
ii Using **c i** or otherwise, find the area between the graph of $f(x) = e^{3x} \sin 2x$ and the x-axis from O to B.

19 A garage is illustrated below on a set of three-dimensional coordinate axes.





right end view



plan view

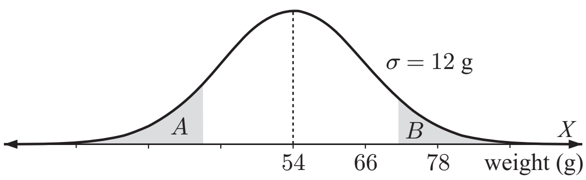
a Find the 3-dimensional coordinates of:
A, B, C, P, Q, R, K, L, M.

b Find the vector equation of [MK].

c Find the acute angle that (MK) makes with (AC).

d An electric light pole has base (6, 2, 0) and is 5 m high. Find point N on [MK] such that N is 5.05 m from the top of the pole.

20 **a** Consider the following normal distribution graph which models the weight of mice.



i Complete the X-scale, and add a Z-scale to the graph.

ii The shaded region A corresponds to 12% of the population. What is the greatest weight of any mouse in the lightest 12% of the population?

iii The shaded region B corresponds to 8% of the population. Find the lowest weight of a mouse in the heaviest 8% of the population.

b A random variable X is normally distributed so that $X \sim N(\mu, \sigma^2)$. If 20% of the values are below 6 and the mean is 9, find σ.

c Whenever Pele takes a penalty shot for goal he has an 80% chance of being successful. If he takes twenty penalty shots at goal, find:

i the expected number of successes

ii the probability that he scores with all but two shots

iii the probability that he scores with at least 18 shots.

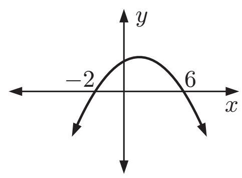
EXAM PRACTICE SET 3 NO CALCULATORS

1 The function f is defined by $f(x) = 2\cos(3x+1)$.
Find: **a** $f''(x)$ **b** $\int f(x)\,dx$

2 State the domain of each of the following functions:

a $y = \sqrt{x-4}$ **b** $y = \sqrt{\dfrac{1}{x-4}}$ **c** $y = \sqrt{\ln(x-4)}$

3 The equation of this curve can be written in the form $y = a(x-p)(x-q)$.



a Write down the values of p and of q.

b Given that the point (4, 3) lies on the curve, find the value of a.

c Write the equation of the curve in the form $y = ax^2 + bx + c$.

4 If A is an obtuse angle and $\sin A = \frac{8}{15}$, calculate $\sin 2A$.

5 Consider the functions $f : x \mapsto 6 - 2x$ and $g : x \mapsto x^2 + 3$.
Find: **a** $f^{-1}(4)$ **b** $(g \circ f)(-2)$

6 Solve:

a $3x^2 + 12x - 5 = 0$

b $2\sin x + \sqrt{3} = 0, \quad 0 \leqslant x \leqslant 2\pi$

7 The coordinates of A, B and C are $(2, -1, 2)$, $(4, -2, 5)$ and $(5, 2, 1)$ respectively.

a Find $\overrightarrow{AB}$.

b Prove that the triangle ABC is right angled at A.

8 Let $\log_b M = x$, $\log_b N = y$ and $\log_b P = z$.

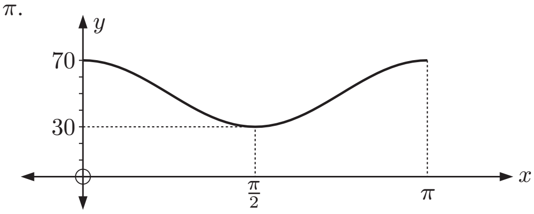
Express $\log_b \dfrac{N^3}{M\sqrt{P}}$ in terms of x, y and z.

9 The velocity of a moving object at time t seconds is given by $v = 30 - 6t^2 \text{ m s}^{-1}$.

a Find its acceleration at the moment when $t = 5$ seconds.

b The initial displacement s is 15 metres. Find an expression for s in terms of t.

10 The diagram shows the graph of $y = a\cos(bx) + c$ for $0 \leqslant x \leqslant \pi$.



Find the values of a, b and c.