SOLUTIONS TO TOPIC 1 (ALGEBRA)

NO CALCULATORS

1 **a** 51, 45, 39, 33, ... is an arithmetic sequence with $u_1 = 51$ and $d = -6$.

$\therefore\ u_{20} = u_1 + 19d$
$= 51 + 19 \times -6$
$= 51 - 114 = -63$

b 0.125, 0.5, 2, 8, ... is a geometric sequence with $u_1 = \frac{1}{8}$ and $r = 4$.

$\therefore\ u_{20} = u_1 r^{19}$
$= \frac{1}{8} \times 4^{19}$
$= 2^{-3} \times 2^{38}$
$= 2^{35}$

2 $u_1 = 27$ and $u_4 = 8$

$\therefore\ u_1 r^3 = 8$
$\therefore\ r^3 = \frac{8}{27}$
$\therefore\ r = \frac{2}{3}$

$\therefore$ the series has sum

$S = \dfrac{u_1}{1-r}$
$= \dfrac{27}{1-\frac{2}{3}}$
$= 81$

3 **a** $u_7 = 1$ and $u_{15} = -23$

$\therefore\ u_1 + 6d = 1$ and $u_1 + 14d = -23$

$\therefore\ u_{15} - u_7 = (u_1 + 14d) - (u_1 + 6d)$
$= -23 - 1$
$\therefore\ 8d = -24$
$\therefore\ d = -3$
and $u_1 = 1 - 6d$
$= 19$
So, $u_{27} = u_1 + 26d$
$= 19 + 26(-3)$
$= -59$

b $S_n = \dfrac{n}{2}(u_1 + u_n)$

$\therefore\ S_{27} = \frac{27}{2}(u_1 + u_{27})$
$= \frac{27}{2}(19 - 59)$
$= -540$

4 **a** $x^2, 3x^2, 5x^2, ...$ is arithmetic with $u_1 = x^2$, $d = 2x^2$.

$\therefore\ u_{10} = u_1 + 9d = x^2 + 9(2x^2)$
$= 19x^2$

b $x^{-\frac{1}{2}}, x, x^{2\frac{1}{2}}, ...$ is geometric with $u_1 = x^{-\frac{1}{2}}$, $r = x^{1\frac{1}{2}}$.

$\therefore\ u_{10} = u_1 r^9$
$= x^{-\frac{1}{2}} \left(x^{\frac{3}{2}}\right)^9$
$= x^{-\frac{1}{2}+\frac{27}{2}}$
$= x^{13}$

5 $u_1 = 18$ and $d = -3$.
If the series has n terms, then

$S_n = -210$
$\therefore\ \dfrac{n}{2}(2u_1 + (n-1)d) = -210$
$\therefore\ \dfrac{n}{2}(2 \times 18 + (n-1) \times (-3)) = -210$
$\therefore\ \dfrac{n}{2}(36 - 3n + 3) = -210$
$n(39 - 3n) = -420$
$\therefore\ 3n^2 - 39n - 420 = 0$
$\therefore\ 3(n^2 - 13n - 140) = 0$
$\therefore\ 3(n-20)(n+7) = 0$
$\therefore\ n = 20\ \{n > 0\}$

So there are 20 terms in the series.

6 **a** $(5x^2)^3 \times \left(\dfrac{x}{4}\right)^2 \times \dfrac{8}{25x^5}$

$= 5^3 x^6 \times \dfrac{x^2}{2^4} \times \dfrac{2^3}{5^2 x^5}$
$= 2^{3-4} 5^{3-2} x^{6+2-5}$
$= \frac{5}{2}x^3$

b $\dfrac{24a^3b^8}{15(a^2b)^3} \div \dfrac{5ab^3}{12a^6b}$

$= \dfrac{24a^3b^8}{15a^6b^3} \times \dfrac{12a^6b}{5ab^3}$
$= \dfrac{\overset{8}{\cancel{24}} \times 12}{\underset{5}{\cancel{15}} \times 5} a^{3-6+6-1} b^{8-3+1-3}$
$= \dfrac{96}{25}a^2b^3$

c $\dfrac{8x^{-2}y^3}{3(xy^2)^0} \times \dfrac{9x^0y^{-1}}{4x^{-3}}$

$= \dfrac{8x^{-2}y^3}{3} \times \dfrac{9y^{-1}}{4x^{-3}}$
$= \dfrac{8 \times 9}{3 \times 4} x^{-2-(-3)} y^{3+(-1)}$
$= 6x^1y^2$
$= 6xy^2$

d $\dfrac{4x^{\frac{1}{2}} \times x^{-1\frac{1}{2}}}{8x^2}$

$= \frac{4}{8}x^{\frac{1}{2}-1\frac{1}{2}-2}$
$= \frac{1}{2}x^{-3}$
$= \dfrac{1}{2x^3}$

e $\sqrt[5]{a^3} \times \sqrt{a^5}$

$= a^{\frac{3}{5}} \times a^{\frac{5}{2}}$
$= a^{\frac{3}{5}+\frac{5}{2}}$
$= a^{\frac{31}{10}}$
$= a^{3.1}$

7 **a** $16^{-\frac{1}{2}}$
$= (4^2)^{-\frac{1}{2}}$
$= 4^{-1}$
$= \frac{1}{4}$

b $81^{\frac{1}{4}}$
$= (3^4)^{\frac{1}{4}}$
$= 3^1$
$= 3$

c $32^{\frac{2}{5}}$
$= (2^5)^{\frac{2}{5}}$
$= 2^2$
$= 4$

d $27^{-\frac{4}{3}}$
$= (3^3)^{-\frac{4}{3}}$
$= 3^{-4}$
$= \frac{1}{3^4}$
$= \frac{1}{81}$

e $\left(\frac{1}{9}\right)^{\frac{3}{2}}$
$= (3^{-2})^{\frac{3}{2}}$
$= 3^{-3}$
$= \frac{1}{3^3}$
$= \frac{1}{27}$

f $(0.008)^{-\frac{5}{3}}$
$= \left(\frac{8}{1000}\right)^{-\frac{5}{3}}$
$= \left(\frac{2}{10}\right)^{3 \times -\frac{5}{3}}$
$= \left(\frac{1}{5}\right)^{-5}$
$= (5^{-1})^{-5}$
$= 5^5$ or 3125

8 **a** $9^x - 6(3^x) + 8$
$= (3^x)^2 - 6(3^x) + 8$
$= (3^x - 4)(3^x - 2)$

b $25^x + 5^{x+1} + 6$
$= (5^x)^2 + 5(5^x) + 6$
$= (5^x + 2)(5^x + 3)$

9 **a** $\log 2 + \log 12$
$= \log(2 \times 12)$
$= \log 24$

b $\log 36 - \log 12$
$= \log\left(\frac{36}{12}\right)$
$= \log 3$

c $3\log 5 + 2\log 3$
$= \log 5^3 + \log 3^2$
$= \log(5^3 \times 3^2)$
$= \log(125 \times 9)$
$= \log 1125$

d $\frac{1}{4}\log 81$
$= \log 81^{\frac{1}{4}}$
$= \log(3^4)^{\frac{1}{4}}$
$= \log 3$

10
$$8^{2x-3} = 16^{2-x}$$
$$\therefore (2^3)^{2x-3} = (2^4)^{2-x}$$
$$\therefore 2^{6x-9} = 2^{8-4x}$$
$$\therefore 6x-9 = 8-4x$$
$$\therefore 10x = 17$$
$$\therefore x = \tfrac{17}{10}$$

11
$$\frac{3^{x+1}-3^x}{2(3^x)-3^{x-1}} = \frac{3^{x-1}(3^2-3)}{3^{x-1}(2\times3-1)} = \tfrac{6}{5}$$

12
$$4^x + 4 = 17(2^{x-1})$$
$$\therefore 2^{2x} - 17(2^{x-1}) + 4 = 0$$
$$\therefore 2\times 2^{2x} - 17(2^x) + 8 = 0$$
$$\therefore 2(2^x)^2 - 17(2^x) + 8 = 0$$
$$\therefore (2(2^x)-1)(2^x-8) = 0$$
$$\therefore 2^x = \tfrac{1}{2} \text{ or } 8$$
$$\therefore 2^x = 2^{-1} \text{ or } 2^3$$
$$\therefore x = -1 \text{ or } 3$$

13 **a**
$$\log_{10}(P^2Q\sqrt{R})$$
$$= \log P^2 + \log Q + \log R^{\frac{1}{2}}$$
$$= 2\log P + \log Q + \tfrac{1}{2}\log R$$
$$= 2A + B + \tfrac{1}{2}C$$

b
$$\log\left(\frac{R^3}{(PQ^2)^4}\right)$$
$$= \log R^3 - \log(PQ^2)^4$$
$$= 3\log R - 4\log(PQ^2)$$
$$= 3\log R - 4(\log P + \log Q^2)$$
$$= 3\log R - 4\log P - 8\log Q$$
$$= 3C - 4A - 8B$$

14
$$\log_5(2x-1) = -1$$
$$\therefore 5^{-1} = 2x-1$$
$$\therefore \tfrac{1}{5} = 2x-1$$
$$\therefore \tfrac{6}{5} = 2x$$
$$\therefore x = \tfrac{3}{5}$$

15
$$\log_b a = \frac{\log_c a}{\log_c b}$$
$$\therefore \log_5 9 = \frac{\log_3 9}{\log_3 5}$$
$$\therefore \frac{8}{\log_5 9} = \frac{8}{\frac{\log_3 9}{\log_3 5}} = \frac{8\log_3 5}{\log_3 9} = \frac{8\log_3 5}{2} = 4\log_3 5$$

16
$$2\ln x + \ln(x-1) - \ln(x-2)$$
$$= \ln x^2 + \ln(x-1) - \ln(x-2)$$
$$= \ln\left(\frac{x^2(x-1)}{(x-2)}\right)$$

17
$$\log_3 x + \log_3(x-2) = 1$$
$$\therefore \log_3(x(x-2)) = 1$$
$$\therefore 3^1 = x(x-2)$$
$$\therefore 3 = x^2 - 2x$$
$$\therefore x^2 - 2x - 3 = 0$$
$$\therefore (x-3)(x+1) = 0$$
$$\therefore x = 3 \quad \{\text{since } x > 2\}$$

18 **a**
$$\log_a(5a) = \log_a 5 + \log_a a = x + 1$$

b
$$\log_a\left(\frac{a^2}{25}\right) = \log_a a^2 - \log_a 25 = 2 - 2\log_a 5 = 2 - 2x$$

19 **a**
$$M = ab^3$$
$$\therefore \log_b M = \log_b(ab^3)$$
$$\therefore \log_b M = \log_b a + \log_b b^3$$
$$\therefore \log_b M = \log_b a + 3$$

b
$$D = \frac{a}{b^2}$$
$$\therefore \log_b D = \log_b\left(\frac{a}{b^2}\right)$$
$$\therefore \log_b D = \log_b a - \log_b b^2$$
$$\therefore \log_b D = \log_b a - 2$$

20 **a**
$$\log_{10} M = 2x - 1$$
$$\therefore 10^{\log_{10} M} = 10^{2x-1}$$
$$\therefore M = 10^{2x-1}$$

b
$$\log_a N = 2\log_a d - \log_a c$$
$$\therefore \log_a N = \log_a d^2 - \log_a c$$
$$\therefore \log_a N = \log_a\left(\frac{d^2}{c}\right)$$
$$\therefore N = \frac{d^2}{c}$$

21

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1 ← 5th row, for $(a+b)^5$

Using the 5th row of Pascal's triangle,
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$
$$\therefore \left(x+\tfrac{1}{x}\right)^5 = x^5 + 5x^4\left(\tfrac{1}{x}\right) + 10x^3\left(\tfrac{1}{x}\right)^2 + 10x^2\left(\tfrac{1}{x}\right)^3 + 5x\left(\tfrac{1}{x}\right)^4 + \left(\tfrac{1}{x}\right)^5$$
$$= x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}$$

CALCULATORS

1
$$u_n = u_1 + (n-1)d = 100 + (n-1)30$$
So, we want to find n where $100 + 30(n-1) > 1200$
$$\therefore 30(n-1) > 1100$$
$$\therefore n-1 > 36.667$$
$$\therefore n > 37.667$$
$\therefore$ the first term is u_{38} which is 1210.

2
$u_5 = u_1r^4 = 18$ and $u_8 = u_1r^7 = 486$
$$\therefore \frac{u_1r^7}{u_1r^4} = \frac{486}{18}$$
$\therefore r^3 = 27$ and so $r = 3$

Since $u_1r^4 = 18$, $u_1 \times 81 = 18$
$$\therefore u_1 = \tfrac{2}{9}$$
So, $u_{12} = u_1r^{11} = \tfrac{2}{9}\times 3^{11} = \dfrac{2\times3^{11}}{3^2} = 2\times3^9$ or $39\,366$

3 **a** Her annual amount is geometric with $u_1 = 800$, $r = 107\% = 1.07$ and $n = 9$.
$$u_9 = u_1 \times r^8 = 800 \times (1.07)^8 \approx 1374.55 \text{ euro}$$

b We need to solve $800 \times (1.07)^n = 4000$

$$\therefore \quad (1.07)^n = 5$$
$$\therefore \quad \log(1.07)^n = \log 5$$
$$\therefore \quad n\log(1.07) = \log 5$$
$$\therefore \quad n = \frac{\log 5}{\log(1.07)}$$
$$\therefore \quad n \approx 23.79$$

So, it will take 24 years.

4 **a** $u_1 = 10$

$u_2 = 10 \times 110\% = 10 \times 1.1$

$u_3 = u_2 \times 1.1 = 10 \times (1.1)^2$

$\vdots$

$u_7 = 10 \times (1.1)^6 = 17.715\,61$

So, Ying ran 17.7 km on day 7.

b Total distance ran $= u_1 + u_2 + u_3 + ... + u_7$ which is geometric with $u_1 = 10,\ r = 1.1,\ n = 7$

$$\therefore \quad S_7 = \frac{10\left((1.1)^7 - 1\right)}{1.1 - 1} = \frac{10\left((1.1)^7 - 1\right)}{0.1} \approx 94.8717$$

So, Ying ran a total of 94.9 km.

5 **a** $10 + 14 + 18 + ... + 138$ is arithmetic with $u_1 = 10,\ d = 4$.

Now $u_1 + (n-1)d = 138$

$\therefore \quad 10 + 4(n-1) = 138$

$\therefore \quad 4(n-1) = 128$

$\therefore \quad n - 1 = 32$

$\therefore \quad n = 33$

So, the sum is

$\frac{n}{2}(u_1 + u_{33})$

$= \frac{33}{2}(10 + 138)$

$= \frac{33}{2}(148)$

$= 2442$

b $6 - 12 + 24 - 48 + 96 - .... + 1536$ is geometric with $u_1 = 6,\ r = -2$.

Now $u_1 r^{n-1} = 1536$

$\therefore \quad 6 \times (-2)^{n-1} = 1536$

$\therefore \quad (-2)^{n-1} = 256$

$\therefore \quad (-2)^{n-1} = (-2)^8$

$\therefore \quad n - 1 = 8$

$\therefore \quad n = 9$

So, the sum is

$\frac{u_1(1 - r^n)}{1 - r}$

$= \frac{6(1 - (-2)^9)}{1 - (-2)}$

$= \frac{6}{3}\left(1 - (-2)^9\right)$

$= 2 \times 513$

$= 1026$

6 **a** $\frac{u_1}{1 - r} = 1.5$

and $u_1 = 1$

$\therefore \quad 1 - r = \frac{1}{1.5}$

$\therefore \quad 1 - r = \frac{2}{3}$

$\therefore \quad r = \frac{1}{3}$

b $S_n = \frac{u_1(1 - r^n)}{1 - r}$

$$\therefore \quad S_7 = \frac{u_1(1 - r^7)}{1 - r} = \frac{1\left(1 - (\frac{1}{3})^7\right)}{1 - \frac{1}{3}} = \frac{3}{2}\left(1 - \frac{1}{2187}\right) = \frac{1093}{729}$$

7 **a** $\frac{u_{n+1}}{u_n} = \frac{12\left(\frac{2}{3}\right)^n}{12\left(\frac{2}{3}\right)^{n-1}}$

$= \frac{2}{3}$ for all $n \in \mathbb{Z}^+$

So, consecutive terms have a common ratio of $\frac{2}{3}$.

Thus, the sequence is geometric with $r = \frac{2}{3}$.

b $u_5 = 12\left(\frac{2}{3}\right)^4 = 12\left(\frac{16}{81}\right) = \frac{64}{27}$

c **i** $\sum_{n=1}^{\infty} u_n = \frac{u_1}{1 - r} = \frac{12\left(\frac{2}{3}\right)^0}{1 - \frac{2}{3}} = \frac{12}{\frac{1}{3}} = 36$

ii $\sum_{n=1}^{20} u_n = S_{20}$

$S_n = \frac{u_1(1 - r^n)}{1 - r}$

$\therefore \quad S_{20} = \frac{12\left(1 - (\frac{2}{3})^{20}\right)}{1 - \frac{2}{3}} \approx 35.9892$

8 Time period $= 33$ months $= 11$ quarters

Interest rate $= 8\%$ p.a. $= 2\%$ per quarter

$\therefore \quad r = 1.02$

$\therefore$ the amount after 11 quarters is

$u_{12} = u_1 \times r^{11}$

$= 3500 \times 1.02^{11}$

≈ 4351.8101

So, the maturing value is £4351.81.

9 **a** $3^x = 243$

$\therefore \quad 3^x = 3^5$

$\therefore \quad x = 5$

b $5 \times 2^x = 160$

$\therefore \quad 2^x = 32$

$\therefore \quad 2^x = 2^5$

$\therefore \quad x = 5$

c $e^x = 27$

$\therefore \quad x = \ln 27$

$\therefore \quad x \approx 3.30$

d $(1.25)^x = 10$

$\therefore \quad \log(1.25)^x = \log 10$

$\therefore \quad x\log(1.25) = 1$

$\therefore \quad x = \frac{1}{\log(1.25)}$

$\therefore \quad x \approx 10.3$

e $7e^x = 100$

$\therefore \quad e^x = \frac{100}{7}$

$\therefore \quad x = \ln\left(\frac{100}{7}\right) \approx 2.66$

10 **a** $200e^{\frac{t}{4}} = 1500$

$\therefore \quad e^{\frac{t}{4}} = 7.5$

$\therefore \quad \frac{t}{4} = \ln(7.5)$

$\therefore \quad t = 4 \times \ln(7.5)$

$t \approx 8.06$

b $0.15e^{0.012t} = 2.18$

$\therefore \quad e^{0.012t} = \frac{2.18}{0.15}$

$\therefore \quad 0.012t = \ln\left(\frac{218}{15}\right)$

$\therefore \quad t = \ln\left(\frac{218}{15}\right) \div 0.012$

$\therefore \quad t \approx 223$

c $20e^{0.2t} = 250$

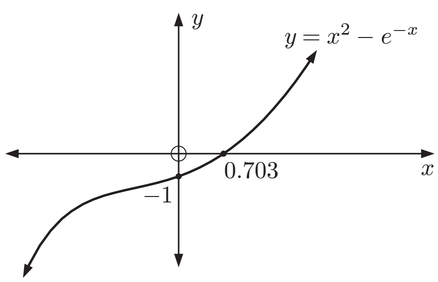
$\therefore \quad e^{0.2t} = \frac{250}{20} = \frac{25}{2}$

$\therefore \quad 0.2t = \ln(12.5)$

$\therefore \quad t = \ln(12.5) \div 0.2$

$\therefore \quad t \approx 12.6$

11 $x^2 > e^{-x} \Leftrightarrow x^2 - e^{-x} > 0$



Using technology, the graph meets the x-axis at $x \approx 0.703$

Hence $x^2 > e^{-x}$ if $x > 0.703$.

12 $\left(x+\dfrac{3}{x^2}\right)^9$ has general term

$$T_{r+1} = \binom{9}{r} x^{9-r}\left(\tfrac{3}{x^2}\right)^r, \quad r = 0, 1, 2, ..., 9$$
$$= \binom{9}{r} x^{9-r} 3^r x^{-2r}$$
$$= \binom{9}{r} x^{9-3r} 3^r$$

The constant term occurs when $9-3r=0$

$\therefore \quad r = 3$

and $T_4 = \binom{9}{3} 3^3 = 2268$

13 $(x+2y^3)^7$ has general term

$$T_{r+1} = \binom{7}{r} x^{7-r}(2y^3)^r, \quad r = 0, 1, 2, ..., 7$$
$$= \binom{7}{r} x^{7-r} 2^r y^{3r}$$

Now when $r=3$, $T_4 = \binom{7}{3} 2^3 x^4 y^9$

$\therefore$ the coefficient of x^4y^9 is $\binom{7}{3} 2^3 = 280$

14 $\left(2x-\dfrac{1}{x^2}\right)^{12}$ has general term

$$T_{r+1} = \binom{12}{r}(2x)^{12-r}\left(\frac{-1}{x^2}\right)^r$$
$$= \binom{12}{r} 2^{12-r} x^{12-r}(-1)^r x^{-2r}$$
$$= \binom{12}{r} 2^{12-r}(-1)^r x^{12-3r}$$

a $12-3r=3$

$\therefore \quad r=3$

$\therefore \quad T_4 = \binom{12}{3} 2^{12-3}(-1)^3 x^3 = -112\,640x^3$

The coefficient of x^3 is $-112\,640$.

b $12-3r=0$

$\therefore \quad r=4$

$\therefore \quad T_5 = \binom{12}{4} 2^{12-4}(-1)^4 x^0 = 126\,720$

The constant term is 126 720.

15 $\left(kx+\dfrac{1}{\sqrt{x}}\right)^9$ has general term

$$T_{r+1} = \binom{9}{r}(kx)^{9-r}\left(\frac{1}{\sqrt{x}}\right)^r$$
$$= \binom{9}{r} k^{9-r} x^{9-r}\frac{1}{x^{\frac{r}{2}}}$$
$$= \binom{9}{r} k^{9-r} x^{9-\frac{3r}{2}}$$

For the constant term, $9-\dfrac{3r}{2}=0$

$\therefore \quad \dfrac{3r}{2} = 9$

$\therefore \quad r=6$

$T_7 = \binom{9}{6} k^3 x^0$

$\therefore \quad 84k^3 = -10\frac{1}{2}$

$\therefore \quad k^3 = -\frac{1}{8}$

$\therefore \quad k = -\frac{1}{2}$

16 $(x+2)(1-x)^{10}$

$$= (x+2)\left(1^{10} + \binom{10}{1}1^9(-x) + ... + \binom{10}{4}1^6(-x)^4 + \binom{10}{5}1^5(-x)^5 + ...\right)$$
$$= (x+2)\left(1-10x+....+\binom{10}{4}x^4 - \binom{10}{5}x^5 + ...\right)$$

So, the terms containing x^5 are $\binom{10}{4}x^5$ and $-2\binom{10}{5}x^5$.

$\therefore$ the coefficient of x^5 is $\binom{10}{4} - 2\binom{10}{5} = -294$

17 If $A = 125\times e^{-kt}$

then $200 = 125e^{-3k}$

$\therefore \quad e^{-3k} = \frac{200}{125}$

$\therefore \quad e^{3k} = \frac{125}{200}$ {reciprocals}

$\therefore \quad 3k = \ln\left(\frac{125}{200}\right)$

$\therefore \quad k = \ln\left(\frac{125}{200}\right)\div 3 \approx -0.157$

18 **a** Pierre added $\$10\times 8 = \$80$

Francesca added $\$(0.50+1+1.5+2+2.5+....+4)$
$= \$18$

b $u_{52} = u_1 + 51d$
$= 0.50 + 51\times 0.50$
$= 26$ So, she added \$26.

c Pierre had $\$10\times 52 + \$100 = \$620$

Francesca had $(0.50+1+1.50+....+26)+100$
$= \frac{52}{2}(0.5+26)+100$
$= 26\times 26.5 + 100$
$= \$789$

19 **a** Hayley: $u_5 = u_1 + 4d = 60 + 4\times 20$
$= 140$ km in week 5

Patrick: $u_5 = u_1 r^4 = 60\times(1.2)^4$
≈ 124 km in week 5

b For Hayley $u_8 = 60 + 7\times 20 = 200$
and $u_9 = 60 + 8\times 20 = 220$

For Patrick $u_7 = 60\times(1.2)^6 \approx 179$
and $u_8 = 60\times(1.2)^7 \approx 215$

So, Patrick is the first to cycle 210 km in a week.

c Hayley: $S_{12} = \frac{12}{2}(2\times 60 + 11\times 20)$
$= 6\times(120+220)$
$= 2040$ km

Patrick: $S_{12} = \dfrac{60\left([1.2]^{12}-1\right)}{1.2-1} \approx 2370$ km

20 **a** 2002: $u_1 = 2000$

2003: $u_2 = 2000 + 2000\times 1.0825$

2004: $u_3 = 2000 + [2000 + 2000\times 1.0825]\times 1.0825$
$= 2000 + 2000r + 2000r^2$ where $r = 1.0825$
$= 2000(1+r+r^2)$

2009: Total amount $= 2000\left(1+r+r^2+r^3+....+r^7\right)$
$$= 2000\left(\frac{r^8-1}{r-1}\right)$$
$$= \frac{2000\left(1.0825^8-1\right)}{0.0825}$$
$\approx 21\,466.32$ rupees

b 2002: $u_1 = 2000$

2009: $2000\times 8 + 7\left(\dfrac{2000\times 9\times 1}{100}\right)$

{8 deposits and 7 years simple interest}
$= 16\,000 + 1260$
$= 17\,260$ rupees

So, Kapil will be 4206.32 rupees better off with the compound interest option.

21 **a** $K(t) = 3200\times(0.85)^t$

$\therefore \quad K(0) = 3200\times 1 = 3200$

So, initially there were 3200 kangaroos.

b $K(5) = 3200 \times (0.85)^5$
≈ 1420 kangaroos

c $N(5) = 2400 + 250 \times 5$
$= 2400 + 1250$
$= 3650$ koalas

d When $K(t) < 1000$

$$3200 \times (0.85)^t < 1000$$
$$\therefore \quad (0.85)^t < \tfrac{1000}{3200}$$
$$\therefore \quad \left(\tfrac{1}{0.85}\right)^t > 3.2 \quad \{\text{reciprocals}\}$$
$$\therefore \quad t \log\left(\tfrac{1}{0.85}\right) > \log(3.2)$$
$$\therefore \quad t > \frac{\log(3.2)}{\log\left(\frac{1}{0.85}\right)} \quad \left\{\log\left(\tfrac{1}{0.85}\right) > 0\right\}$$
$$\therefore \quad t > 7.1570$$

So, the kangaroo population falls below 1000 about 7 years 2 months after the start of 2000, which is the end of February 2007.

e We need to solve $2400 + 250t > 3200 \times (0.85)^t$.
Using technology, $t \approx 1.102$.
So, the number of koalas exceeds the number of kangaroos about 1 year 1 month after the start of 2000, which is at the start of February 2001.

SOLUTIONS TO TOPIC 2 (FUNCTIONS AND EQUATIONS)

NO CALCULATORS

1 **a** No, as a vertical line can cut the graph more than once.
b Yes **c** Yes **d** No **e** No **f** Yes

2 **a** $G(-1)$
$= 3 - (-1) - 2(-1)^2$
$= 3 + 1 - 2$
$= 2$

b $G(3)$
$= 3 - (3) - 2(3)^2$
$= 3 - 3 - 18$
$= -18$

c $G(a^3)$
$= 3 - (a^3) - 2(a^3)^2$
$= 3 - a^3 - 2a^6$

d $G(a-2)$
$= 3 - (a-2) - 2(a-2)^2$
$= 3 - a + 2 - 2[a^2 - 4a + 4]$
$= -3 + 7a - 2a^2$

3 $f(x) = 5x - 2, \quad g(x) = 2x + 7$

a $(f \circ g)(x)$
$= f(g(x))$
$= f(2x+7)$
$= 5(2x+7) - 2$
$= 10x + 33$

b $(g \circ f)(x)$
$= g(f(x))$
$= g(5x-2)$
$= 2(5x-2) + 7$
$= 10x + 3$

c g is $y = 2x + 7 \quad \therefore \quad g^{-1}$ is $x = 2y + 7$
or $x - 7 = 2y$
or $y = \dfrac{x-7}{2}$
So, $g^{-1}(x) = \dfrac{x-7}{2}$

4 Domain $= \{x \mid x \geqslant -1, x \neq 1\}$

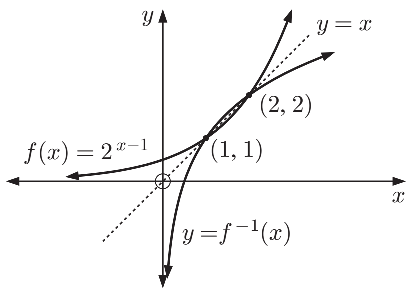
5 Domain $= \{x \mid x > 2\}$, Range $= \{y \mid y \in \mathbb{R}\}$

6 **a** $f(g(x)) = f(4-x)$
$= 3(4-x) + 1$
$= -3x + 13$

b $(g \circ f)(x) = g(3x+1)$
$= 4 - (3x+1)$
$= 3 - 3x$
$\therefore \quad (g \circ f)(-4) = 3 - 3(-4)$
$= 15$

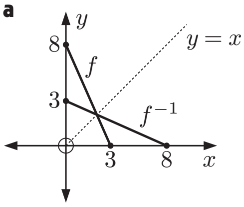
c f is $y = 3x + 1$, so f^{-1} is $x = 3y + 1$
$\therefore \quad y = \dfrac{x-1}{3}$
$\therefore \quad f^{-1}(x) = \dfrac{x-1}{3}$
so $f^{-1}(\tfrac{1}{2}) = \dfrac{\frac{1}{2} - 1}{3} = -\tfrac{1}{6}$

7 **a**

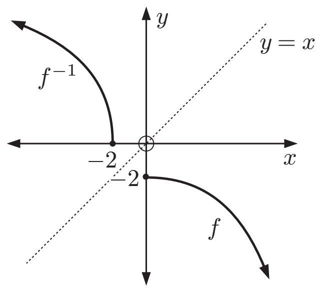


b $f(x)$ is $y = 2^{x-1}$
so $f^{-1}(x)$ is $x = 2^{y-1}$
$\therefore \quad \log x = (y-1)\log 2$
$\therefore \quad \dfrac{\log x}{\log 2} = y - 1$
$\therefore \quad y = \log_2 x + 1$
$\therefore \quad f^{-1}(x) = \log_2 x + 1$

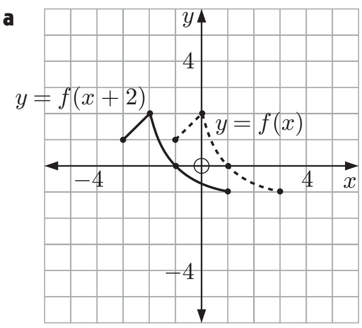
8 **a**



b



9 **a**



b