# Chapter 13

## LINES AND PLANES IN SPACE

### EXERCISE 13A.1

**1** **a** **i** $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3\\-4\end{pmatrix} + t\begin{pmatrix}1\\4\end{pmatrix}, \quad t \in \mathbb{R}$

**ii** $x = 3 + t, \quad y = -4 + 4t$

$\therefore \quad t = x - 3 = \dfrac{y+4}{4}$

$\therefore \quad 4x - 12 = y + 4$

$\therefore \quad 4x - y = 16$

**b** **i** If the line has direction vector **b** perpendicular to $\begin{pmatrix}-8\\2\end{pmatrix}$, then

$\mathbf{b} \bullet \begin{pmatrix}-8\\2\end{pmatrix} = 0$

$\therefore \quad \mathbf{b} = \begin{pmatrix}2\\8\end{pmatrix}$ is a reasonable choice

$\therefore \quad \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}5\\2\end{pmatrix} + t\begin{pmatrix}2\\8\end{pmatrix}, \quad t \in \mathbb{R}$

**ii** $x = 5 + 2t, \quad y = 2 + 8t$

$\therefore \quad t = \dfrac{x-5}{2} = \dfrac{y-2}{8}$

$\therefore \quad 8x - 40 = 2y - 4$

$\therefore \quad 8x - 2y = 36$

$\therefore \quad 4x - y = 18$

**c** **i** $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-6\\0\end{pmatrix} + t\begin{pmatrix}3\\7\end{pmatrix}, \quad t \in \mathbb{R}$

**ii** $x = -6 + 3t, \quad y = 7t$

$\therefore \quad t = \dfrac{x+6}{3} = \dfrac{y}{7}$

$\therefore \quad 7x + 42 = 3y$

$\therefore \quad 7x - 3y = -42$

**d** **i** Take $(-1, 11)$ as our fixed point, so $\mathbf{a} = \begin{pmatrix}-1\\11\end{pmatrix}$.

The direction vector $\mathbf{b} = \begin{pmatrix}-3-(-1)\\12-11\end{pmatrix}$

$= \begin{pmatrix}-2\\1\end{pmatrix}$.

$\therefore \quad \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-1\\11\end{pmatrix} + t\begin{pmatrix}-2\\1\end{pmatrix}, \quad t \in \mathbb{R}$

**ii** $x = -1 - 2t, \quad y = 11 + t$

$\therefore \quad t = \dfrac{x+1}{-2} = y - 11$

$\therefore \quad x + 1 = -2y + 22$

$\therefore \quad x + 2y = 21$

**2** $x = -1 + 2t, \; y = 4 - t, \quad t \in \mathbb{R}$

When $t = 0$, $x = -1 + 2(0) = -1$ and $y = 4 - 0 = 4$ $\quad\therefore$ the point is $(-1, 4)$.

When $t = 1$, $x = -1 + 2(1) = 1$ and $y = 4 - 1 = 3$ $\quad\therefore$ the point is $(1, 3)$.

When $t = 3$, $x = -1 + 2(3) = 5$ and $y = 4 - 3 = 1$ $\quad\therefore$ the point is $(5, 1)$.

When $t = -1$, $x = -1 + 2(-1) = -3$ and $y = 4 - -1 = 5$ $\quad\therefore$ the point is $(-3, 5)$.

When $t = -4$, $x = -1 + 2(-4) = -9$ and $y = 4 - -4 = 8$ $\quad\therefore$ the point is $(-9, 8)$.

**3** **a** If $t + 2 = 3$ and $1 - 3t = -2$,

then $t = 1$ and $-3t = -3$

$\therefore \quad t = 1$

Since $t = 1$ in each case, $(3, -2)$ lies on the line.

**b** If $(k, 4)$ lies on $x = 1 - 2t, \quad y = 1 + t$, then

$k = 1 - 2t$ and $4 = 1 + t$

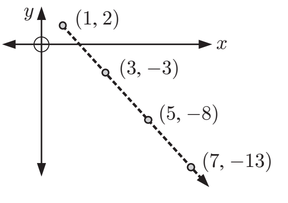
$\therefore \quad t = 3$

and $k = 1 - 6 = -5$

**4** **a** $x(0) = 1$ and $y(0) = 2$,

$\therefore$ the initial position is $(1, 2)$

**b**




**c** In 1 second, the $x$-step is 2 and $y$-step is $-5$, which is a distance of $\sqrt{2^2 + (-5)^2} = \sqrt{29}$ cm

$\therefore$ the speed is $\sqrt{29}$ cm s$^{-1}$.

**5** In parametric form: $x = 1 - t, \quad y = 5 + 3t, \quad t \in \mathbb{R}$

In Cartesian form: $t = \dfrac{x-1}{-1} = \dfrac{y-5}{3}$

$\therefore \quad 3x - 3 = -y + 5$

$\therefore \quad 3x + y = 8$

## EXERCISE 13A.2

**1** **a** The vector equation is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad t \in \mathbb{R}$

**b** The vector equation is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \quad t \in \mathbb{R}$

**c** Since the line is parallel to the $X$-axis, it has direction vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\therefore$ the vector equation is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad t \in \mathbb{R}$

**2** **a** The parametric equations are:

$x = 5 + (-1)t, \;\; y = 2 + 2t, \;\; z = -1 + 6t$

$\therefore \quad x = 5 - t, \;\; y = 2 + 2t, \;\; z = -1 + 6t, \;\; t \in \mathbb{R}$

**b** The parametric equations are:

$x = 0 + 2t, \;\; y = 2 + (-1)t, \;\; z = -1 + 3t$

$\therefore \quad x = 2t, \;\; y = 2 - t, \;\; z = -1 + 3t, \;\; t \in \mathbb{R}$

**c** Since the line is perpendicular to the $XOY$ plane, it has direction vector $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

$\therefore$ the parametric equations are: $x = 3 + 0t, \;\; y = 2 + 0t, \;\; z = -1 + 1t$

$\therefore \quad x = 3, \;\; y = 2, \;\; z = -1 + t, \;\; t \in \mathbb{R}$

**3** **a** $\overrightarrow{AB} = \begin{pmatrix} -1-1 \\ 3-2 \\ 2-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \quad \therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R}$

**b** $\overrightarrow{CD} = \begin{pmatrix} 3-0 \\ 1-1 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \quad \therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}, \quad t \in \mathbb{R}$

**c** $\overrightarrow{EF} = \begin{pmatrix} 1-1 \\ -1-2 \\ 5-5 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} \quad \therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}, \quad t \in \mathbb{R}$

**d** $\overrightarrow{GH} = \begin{pmatrix} 5-0 \\ -1-1 \\ 3--1 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} \quad \therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + t\begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}, \quad t \in \mathbb{R}$

**4** Given $x = 1 - t, \;\; y = 3 + t, \;\; z = 3 - 2t$:

**a** The line meets the $XOY$ plane when $z = 0$ $\quad \therefore \; 3 - 2t = 0$

$\therefore \; t = \frac{3}{2}$

Then $x = 1 - \frac{3}{2} = -\frac{1}{2}$ and $y = 3 + \frac{3}{2} = \frac{9}{2}$, so the point is $\left(-\frac{1}{2}, \frac{9}{2}, 0\right)$.

**b** The line meets the $YOZ$ plane when $x = 0$ $\quad \therefore \; 1 - t = 0$

$\therefore \; t = 1$

Then $y = 3 + 1 = 4$ and $z = 3 - 2 = 1$, so the point is $(0, 4, 1)$.

**c** The line meets the $XOZ$ plane when $y = 0$ $\therefore\ 3 + t = 0$

$\therefore\ t = -3$

Then $x = 1 - (-3) = 4$ and $z = 3 - 2(-3) = 9$, so the point is $(4, 0, 9)$.

**5** Given a line with equations $x = 2 - t$, $y = 3 + 2t$ and $z = 1 + t$,

the distance to the point $(1, 0, -2)$ is $\sqrt{(2 - t - 1)^2 + (3 + 2t - 0)^2 + (1 + t + 2)^2}$.

But this distance $= 5\sqrt{3}$ units

$$\therefore\ \sqrt{(1-t)^2 + (3+2t)^2 + (t+3)^2} = 5\sqrt{3}$$
$$\therefore\ (1-t)^2 + (3+2t)^2 + (t+3)^2 = 75$$
$$\therefore\ 1 - 2t + t^2 + 9 + 12t + 4t^2 + t^2 + 6t + 9 = 75$$
$$\therefore\ 6t^2 + 16t - 56 = 0$$
$$\therefore\ 3t^2 + 8t - 28 = 0$$
$$\therefore\ (3t + 14)(t - 2) = 0$$
$$\therefore\ t = -\tfrac{14}{3} \text{ or } t = 2$$

When $t = 2$ the point is $(0, 7, 3)$, and when $t = -\frac{14}{3}$ the point is $\left(\frac{20}{3}, -\frac{19}{3}, -\frac{11}{3}\right)$.

## EXERCISE 13A.3

**1** $L_1$ has direction vector $\binom{4}{-3}$ and $L_2$ has direction vector $\binom{5}{4}$. If $\theta$ is the angle between them,

$$\cos\theta = \frac{\left|\binom{4}{-3} \bullet \binom{5}{4}\right|}{\sqrt{16+9}\sqrt{25+16}} = \frac{|20 + (-12)|}{\sqrt{25 \times 41}} = \frac{8}{\sqrt{25 \times 41}}$$

$$\therefore\ \theta = \cos^{-1}\left(\frac{8}{\sqrt{25 \times 41}}\right) \approx 75.5^o$$

$\therefore$ the required angle measures $75.5^o$.

**2** $L_1$ has direction vector $\binom{12}{5}$ and $L_2$ has direction vector $\binom{3}{-4}$. If $\theta$ is the angle between them,

$$\cos\theta = \frac{\left|\binom{12}{5} \bullet \binom{3}{-4}\right|}{\sqrt{144+25}\sqrt{9+16}} = \frac{|36 + (-20)|}{13 \times 5} = \tfrac{16}{65}$$

$$\therefore\ \theta = \cos^{-1}\left(\tfrac{16}{65}\right) \approx 75.7^o$$

**3** Line 1 has direction vector $\binom{5}{-2}$ and line 2 has direction vector $\binom{4}{10}$

and $\binom{5}{-2} \bullet \binom{4}{10} = 20 + (-20) = 0$

$\therefore$ the lines are perpendicular.

**4** **a** Line 1 has direction vector $\begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix}$ and line 2 has direction vector $\begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}$.

If $\theta$ is the angle between them,

$$\cos\theta = \frac{\left|\begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}\right|}{\sqrt{9+256+49}\sqrt{9+64+25}} = \frac{|9 - 128 - 35|}{\sqrt{314}\sqrt{98}} = \frac{154}{\sqrt{314 \times 98}}$$

$\therefore\ \theta \approx 28.6^o$

**b** Since $L_1 \perp L_3$, $\begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ -3 \\ x \end{pmatrix} = 0$

$$\therefore\ 48 + 7x = 0$$
$$\therefore\ x = -\tfrac{48}{7}$$

## EXERCISE 13B.1

**1** **a** **i** When $t = 0$, $\binom{x}{y} = \binom{-4}{3}$
$\therefore$ the object is at $(-4, 3)$.

**ii** The velocity vector is $\binom{12}{5}$.

**iii** The speed is $\sqrt{12^2 + 5^2}$
$= 13\ \text{m s}^{-1}$

**b** **i** When $t = 0$, $\binom{x}{y} = \binom{0}{-6}$
$\therefore$ the object is at $(0, -6)$.

**ii** The velocity vector is $\binom{3}{-4}$.

**iii** The speed is $\sqrt{3^2 + (-4)^2}$
$= 5\ \text{m s}^{-1}$

**c** **i** When $t = 0$, $\binom{x}{y} = \binom{-2}{-7}$
$\therefore$ the object is at $(-2, -7)$.

**ii** The velocity vector is $\binom{-6}{-4}$.

**iii** The speed is $\sqrt{(-6)^2 + (-4)^2}$
$= \sqrt{52}\ \text{m s}^{-1}$

**d** **i** When $t = 0$,
$x = 5$ and $y = -5$
$\therefore$ the object is at $(5, -5)$.

**ii** The velocity vector is $\binom{8}{4}$.

**iii** The speed is $\sqrt{8^2 + 4^2}$
$= \sqrt{80}\ \text{m s}^{-1}$

**2** **a** $\binom{4}{-3}$ has length $\sqrt{4^2 + (-3)^2} = 5$
$\therefore$ $30\binom{4}{-3}$ has length 150
$\therefore$ the velocity vector is $\binom{120}{-90}$.

**b** $\binom{24}{7}$ has length $\sqrt{24^2 + 7^2} = 25$
$\therefore$ $\frac{1}{2}\binom{24}{7}$ has length 12.5
$\therefore$ the velocity vector is $\binom{12}{3.5}$.

**c** $2\mathbf{i} + \mathbf{j} = \binom{2}{1}$ has length $\sqrt{2^2 + 1^2} = \sqrt{5}$
$\therefore$ $10\sqrt{5}\binom{2}{1}$ has length 50
$\therefore$ the velocity vector is $\binom{20\sqrt{5}}{10\sqrt{5}}$.

**3** $-2\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}$ has length $\sqrt{(-2)^2 + 5^2 + (-14)^2} = \sqrt{4 + 25 + 196} = \sqrt{225} = 15$

$\therefore$ $6\begin{pmatrix} -2 \\ 5 \\ -14 \end{pmatrix}$ has length 90, so the velocity vector is $\begin{pmatrix} -12 \\ 30 \\ -84 \end{pmatrix}$.

## EXERCISE 13B.2

**1** **a** $\mathbf{r} = \mathbf{a} + t\mathbf{b}$
$\therefore$ $\binom{x}{y} = \binom{-3}{-2} + t\binom{2}{4}$, $t \geqslant 0$
$\therefore$ $\binom{x}{y} = \binom{-3+2t}{-2+4t}$

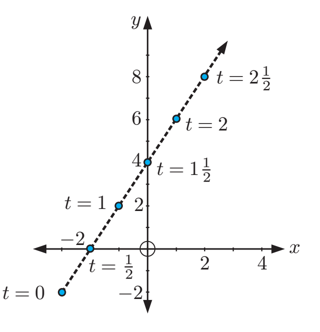
**b** At $t = 2.5$, $-3 + 2t = -3 + 5 = 2$
and $-2 + 4t = -2 + 10 = 8$

So, the position vector is $\binom{2}{8}$.

**c** **i** When the car is due north, $x = 0$.
$\therefore$ $-3 + 2t = 0$
$\therefore$ $t = 1.5$ seconds

**ii** When the car is due west, $y = 0$.
$\therefore$ $-2 + 4t = 0$
$\therefore$ $t = 0.5$ seconds

**d**




**2** Yacht A: $\binom{x_A}{y_B} = \binom{4}{5} + t\binom{1}{-2}$ Yacht B: $\binom{x_B}{y_B} = \binom{1}{-8} + t\binom{2}{1}$, $t \geqslant 0$

**a** When $t = 0$, $\binom{x_A}{y_A} = \binom{4}{5}$ $\therefore$ A is at $(4, 5)$
and $\binom{x_B}{y_B} = \binom{1}{-8}$ $\therefore$ B is at $(1, -8)$.

**b** For A, the velocity vector is $\binom{1}{-2}$, and for B it is $\binom{2}{1}$.

**c** Speed of A $= \sqrt{1^2 + (-2)^2} = \sqrt{5}\ \text{km h}^{-1}$. Speed of B $= \sqrt{2^2 + 1^2} = \sqrt{5}\ \text{km h}^{-1}$.

**d** A has direction vector $\begin{pmatrix}1\\-2\end{pmatrix}$ and B has direction vector $\begin{pmatrix}2\\1\end{pmatrix}$.

Since $\begin{pmatrix}1\\-2\end{pmatrix} \bullet \begin{pmatrix}2\\1\end{pmatrix} = 2-2 = 0$, the paths of the yachts are at right angles to each other.

**3** **a** P's torpedo has position $\begin{pmatrix}x_1\\y_1\end{pmatrix} = \begin{pmatrix}-5\\4\end{pmatrix} + t\begin{pmatrix}3\\-1\end{pmatrix}$ and at $t=0$, the time is 1:34 pm

$\therefore \quad x_1(t) = -5+3t, \quad y_1(t) = 4-t.$

**b** Speed $= \sqrt{3^2+(-1)^2} = \sqrt{10}$ km min$^{-1}$

**c** Q fires its torpedo after $a$ minutes.

$\therefore$ at time $t$, its torpedo has travelled for $(t-a)$ minutes.

$\therefore \quad \begin{pmatrix}x_2\\y_2\end{pmatrix} = \begin{pmatrix}15\\7\end{pmatrix} + (t-a)\begin{pmatrix}-4\\-3\end{pmatrix}, \quad t \geqslant a$

$\therefore \quad x_2(t) = 15-4(t-a)$ and $y_2(t) = 7-3(t-a)$

**d** They meet when $x_1(t) = x_2(t)$ and $y_1(t) = y_2(t)$

$\therefore \quad -5+3t = 15-4(t-a)$ and $4-t = 7-3(t-a)$

$\therefore \quad 7t-4a = 20$ .... (1) and $2t-3a = 3$ .... (2)

Solving simultaneously,
$$\begin{aligned} 21t - 12a &= 60 && \{3\times(1)\}\\ -8t+12a &= -12 && \{-4\times(2)\}\\ \text{adding} \quad 13t \quad &= 48 \end{aligned}$$

$\therefore \quad t = \frac{48}{13}$ and $7\left(\frac{48}{13}\right) - 4a = 20$

$\therefore \quad t \approx 3.6923$ $\qquad \therefore \quad 5.8462 = 4a$

$\therefore \quad t \approx 3$ min 41.54 sec $\qquad \therefore \quad a \approx 1.4615 \approx 1$ min 27.7 sec

So, as $a \approx 1.4615$, Q fired at 1:35:28 pm, and the explosion occurred at 1:37:42 pm.

**4** **a** $\overrightarrow{AB} = \begin{pmatrix}3-6\\10-9\\2.5-3\end{pmatrix} = \begin{pmatrix}-3\\1\\-0.5\end{pmatrix}$

**b** $|\overrightarrow{AB}| = \sqrt{(-3)^2+1^2+(-0.5)^2}$

$= \sqrt{10.25}$ km

The helicopter travels $\sqrt{10.25}$ km in 10 minutes.

$\therefore$ the helicopter's speed is

$6\times\sqrt{10.25} \approx 19.2$ km h$^{-1}$.

**c** $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\9\\3\end{pmatrix} + t\begin{pmatrix}-3\\1\\-0.5\end{pmatrix}, \quad t\in\mathbb{R}$

**d** If $z=0$, $3+(-0.5)t = 0$

$\therefore \quad t = 6$

$t=1$ represents 10 minutes, so $t=6$ represents 60 minutes.

$\therefore$ the helicopter lands on the helipad after 1 hour.

## EXERCISE 13B.3

**1** **a** $6\mathbf{i} - 6\mathbf{j}$

**b** The length of $\begin{pmatrix}-3\\4\end{pmatrix} = \sqrt{(-3)^2+4^2} = \sqrt{25} = 5$

As the speed is 10 km h$^{-1}$, the liner has velocity vector $2\begin{pmatrix}-3\\4\end{pmatrix} = \begin{pmatrix}-6\\8\end{pmatrix}$.

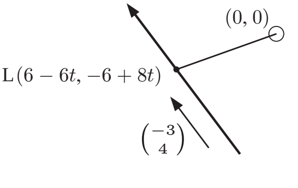
$\therefore$ the liner has position $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}6\\-6\end{pmatrix} + t\begin{pmatrix}-6\\8\end{pmatrix}, \quad t\geqslant 0, \quad t$ in hours.

**c** The liner is due east when $y=0$

$\therefore \quad -6+8t = 0$

$\therefore$ at $t = \frac{3}{4}$ hour

**d** The liner L is nearest the fishing boat O when $\overrightarrow{OL} \perp \begin{pmatrix}-3\\4\end{pmatrix}$.




$\therefore \quad \overrightarrow{OL} \bullet \begin{pmatrix}-3\\4\end{pmatrix} = 0$

$\therefore \quad \begin{pmatrix}6-6t\\-6+8t\end{pmatrix} \bullet \begin{pmatrix}-3\\4\end{pmatrix} = 0$

$\therefore\ (-18+18t)+(-24+32t)=0$

$\therefore\ 50t=42$

$\therefore\ t=0.84$ hours $=50.4$ minutes

and when $t=0.84$, $\text{L}=\begin{pmatrix}6-6(0.84)\\-6+8(0.84)\end{pmatrix}=\begin{pmatrix}0.96\\0.72\end{pmatrix}$

$\therefore$ the liner is closest to the fishing boat after 0.84 hours or 50.4 minutes, when it is at (0.96, 0.72).

**2** **a** $|\mathbf{b}|=\sqrt{(-3)^2+(-1)^2}=\sqrt{10}$

As the speed is $40\sqrt{10}$ km h$^{-1}$, the velocity vector is $40\binom{-3}{-1}=\binom{-120}{-40}$.

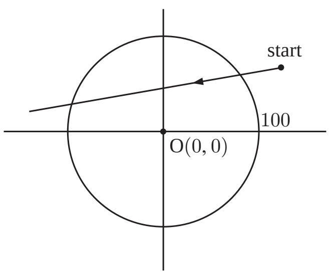
**b** $\binom{x}{y}=\binom{200}{100}+t\binom{-120}{-40}$, $t\geqslant 0$ $\{t=0$ at 12:00 noon$\}$

**c** At 1:00 pm, $t=1$ and $\binom{x}{y}=\binom{200-120}{100-40}=\binom{80}{60}$

**d** The distance from O(0, 0) to $\text{P}_1(80, 60)$ is $\left|\binom{80}{60}\right|=\sqrt{80^2+60^2}=100$ km,

which is when it becomes visible to radar. {within 100 km of O(0, 0)}

**e**




A general point on the path is P$(200-120t, 100-40t)$.

Now $\overrightarrow{\text{OP}}=\binom{200-120t}{100-40t}$,

and for the closest point $\overrightarrow{\text{OP}}\bullet\binom{-3}{-1}=0$

$\therefore\ -3(200-120t)-1(100-40t)=0$

$\therefore\ -700+400t=0$

$\therefore\ t=\frac{7}{4}=1\frac{3}{4}$ hours

$\therefore$ the time when the aircraft is closest is 1:45 pm, and

at this time $\overrightarrow{\text{OP}}=\begin{pmatrix}200-120(\frac{7}{4})\\100-40(\frac{7}{4})\end{pmatrix}=\binom{-10}{30}$

$\therefore\ d_{\min}=\sqrt{(-10)^2+30^2}\approx 31.6$ km

**f** It disappears from radar when $|\overrightarrow{\text{OP}}|=100$ and $t>1\frac{3}{4}$

$\therefore\ \sqrt{(200-120t)^2+(100-40t)^2}=100$

$\therefore\ 40\,000-48\,000t+14\,400t^2+10\,000-8000t+1600t^2=10\,000$

$\therefore\ 16\,000t^2-56\,000t+40\,000=0$

$\therefore\ 16t^2-56t+40=0$ $\{\div 1000\}$

$\therefore\ 2t^2-7t+5=0$ $\{\div 8\}$

$\therefore\ (2t-5)(t-1)=0$

$\therefore\ t=\frac{5}{2}$ {as $t>1\frac{3}{4}$}

So, the aircraft disappears from the radar screen $2\frac{1}{2}$ hours after noon, or at 2:30 pm.

**3** **a** At A, $y=0$

$\therefore\ 2x=36$

$\therefore\ x=18$

At B, $x=0$

$\therefore\ 3y=36$

$\therefore\ y=12$

So A is (18, 0) and B is (0, 12).

**b** $2x+3y=36$

$\therefore\ 3y=36-2x$

$\therefore\ y=\dfrac{36-2x}{3}$

$\therefore$ any point R on the railway track can be written $\text{R}\left(x, \dfrac{36-2x}{3}\right)$.

**c** $\overrightarrow{\text{PR}}=\begin{pmatrix}x-4\\\frac{36-2x}{3}-0\end{pmatrix}=\begin{pmatrix}x-4\\\frac{36-2x}{3}\end{pmatrix}$

$\overrightarrow{\text{AB}}=\binom{0-18}{12-0}=\binom{-18}{12}$