Investigation and modelling questions 34

Contents:

Many of the questions in **Chapters 33** and **34** are adapted from past examination papers for IGCSE Mathematics 0580 by permission of the University of Cambridge Local Examinations Syndicate. The 0580 course is a different syllabus from that followed by students of the 0607 course, but has many features in common. These questions are certainly appropriate for practising mathematical techniques and applications relevant to the 0607 curriculum, but do not necessarily represent the style of question that will be encountered on the 0607 examination papers. Teachers are referred to the specimen papers of the 0607 syllabus for a more representative group of questions. The University of Cambridge Local Examinations Syndicate bears no responsibility for the example answers to questions taken from its past question papers which are contained in this publication.

A INVESTIGATION QUESTIONS

1 It is given that $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + ... + n^2 = \dfrac{n(n+1)(2n+1)}{k}$ where $k \in \mathbb{Z}$.

a If $n = 1$, the LHS is 1^2.
If $n = 2$, the LHS is $1^2 + 2^2$.
Use $n = 1$ to find the value of k, and check that you get the same value of k for $n = 2$ and $n = 3$.

b Use the given formula to find the value of $1^2 + 2^2 + 3^2 + 4^2 + ... + 100^2$.

c Notice that
$$\begin{aligned} 2^2 + 4^2 + 6^2 &= (2 \times 1)^2 + (2 \times 2)^2 + (2 \times 3)^2 \\ &= 2^2 1^2 + 2^2 2^2 + 2^2 3^2 \\ &= 2^2(1^2 + 2^2 + 3^2) \end{aligned}$$

Hence,

i find m if $2^2 + 4^2 + 6^2 + 8^2 + ... + 100^2 = 2^2(1^2 + 2^2 + 3^2 + ... + m^2)$

ii find the value of $2^2 + 4^2 + 6^2 + 8^2 + ... + 100^2$.

d Use **b** and **c ii** to find the value of $1^2 + 3^2 + 5^2 + 7^2 + ... + 99^2$.

e Use some of the previous answers to find the value of:
$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + ... + 99^2 - 100^2.$$

2 A farmer makes a sheep pen in the shape of a quadrilateral from four pieces of fencing. Each side of the quadrilateral is 5 metres long and one of the angles is 60^o.

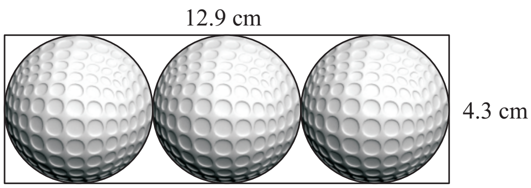
- **a** Using a scale of 1 to 100, make an accurate drawing of the quadrilateral.
- **b** Mark in its axes of symmetry with broken lines and describe how they cut each other.
- **c** What is the special geometrical name of this shape?
- **d** Calculate the area enclosed by the sheep pen, giving your answer in square metres.
- **e** By changing the angles (but leaving the lengths of the sides unchanged), the area enclosed by the sheep pen can be varied. What is the greatest possible area that can be enclosed? Justify your answer.

3 Throughout this question, remember that 1 is *not* a prime number.

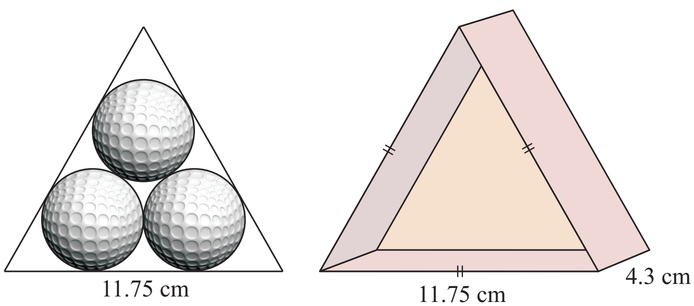
- **a** Find a prime number which can be written as the sum of two prime numbers.
- **b** Consider the statement "All even numbers greater than 15 can be written as the sum of two different prime numbers in at least two different ways." For example, $20 = 3 + 17 = 7 + 13$.
  - **i** Show that the above statement is true for 16.
  - **ii** Find a number between 30 and 50 which shows that the statement is false.
- **c** Show that 16 can be written as the sum of three different prime numbers.
- **d** Consider the statement "All odd numbers greater than 3 can be written as the sum of two prime numbers". Is this statement true or false? Justify your answer.

4 **Adapted from June 1989, Paper 4**

A firm which manufactures golf balls is experimenting with the packaging of its product. 3 golf balls, each of radius 2.15 centimetres, are packaged in a rectangular box, a cross-section of which is shown in the diagram alongside. The box is 12.9 centimetres long, 4.3 centimetres wide and 4.3 centimetres high.



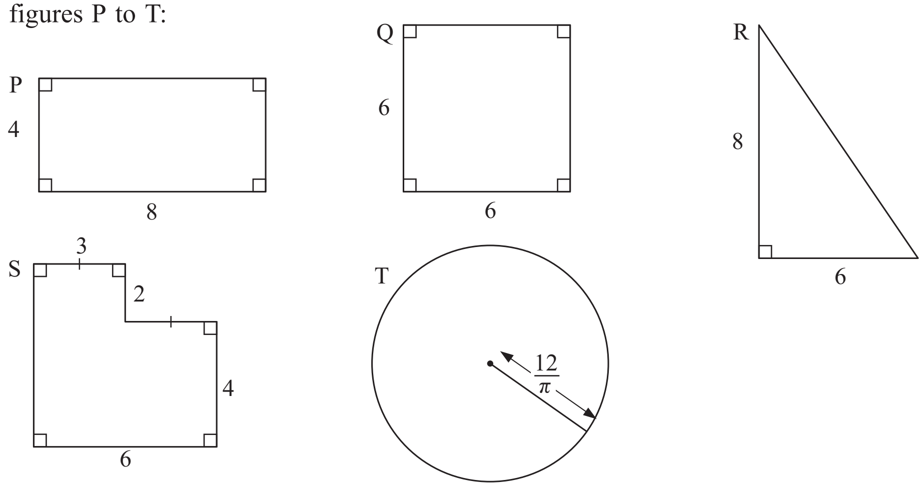
- **a** Given that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$, calculate the amount of space within the box which is unfilled.
- **b** The marketing department suggests that an equilateral triangular box of side 11.75 centimetres and height 4.3 centimetres might be more attractive. The diagrams show a plan and side view of the new box.



  Calculate the amount of space within this new box which is unfilled.
- **c** Give your answer to **a** and **b** as percentages of the capacity of each container.
- **d** Design a box of your own which gives a smaller percentage of unfilled space.

5 Adapted from June 1989, Paper 6

Consider the figures P to T:



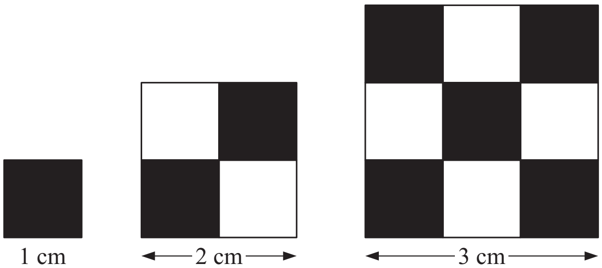
a All five figures have something important in common. What is it?

b Calculate the area of a regular hexagon (H) of side 4 centimetres.

c Using the letters P, Q, R, S, T and H, list the areas in order of size, starting with the smallest.

d Explain any conclusions you arrive at.

6 June 1988, Specimen Paper 6



The diagram shows 3 squares, the sides of which are 1 cm, 2 cm and 3 cm respectively. Each of the small squares on the diagram has a side of length 1 cm and alternate squares are coloured black and white.

a The number of small squares of each colour used is shown in the following table. Copy and complete the table.

Length of side of given square	L	1	2	3	4	5
Number of black squares	B	1	2	5		
Number of white squares	W	0	2	4		
Total number of squares	T	1	4	9		

b **i** How many small white squares will there be when a square of side 11 cm is drawn?

ii Find the length of the side of a square when 1681 small black and white squares are needed to cover it.

c Write down a formula connecting T and L.

d Write down a formula connecting T and B when

i B is an even number **ii** B is an odd number.

7 Adapted from June 1989, Paper 6

a Copy and complete the following two sets of calculations.

1	$=$	1^3	$=$
$1+2$	$=$	1^3+2^3	$=$
$1+2+3$	$=$	$1^3+2^3+3^3$	$=$
$1+2+3+4$	$=$	$1^3+2^3+3^3+4^3$	$=$

b How are the two sets of results related?

c Find the value of $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3$.

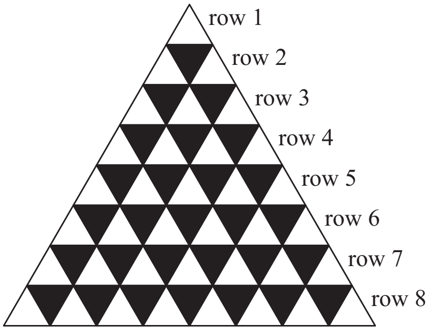
d Given that the sum of the first 25 numbers, $1+2+3+\ldots\ldots+25$, is 325, find the value of $1^3+2^3+3^3+\ldots\ldots+25^3$.

e $1+2+3+4+5+\ldots\ldots+n = an^2+bn$. Find the values of a and b, and test your answers.

f Find the value of $1^3+2^3+3^3+4^3+\ldots\ldots+250^3$.

8 June 1988, Paper 6

The diagram shows the first eight rows of a continuing pattern of black and white triangles.



a Find a formula for each of the following:

i the number of triangles in the nth row

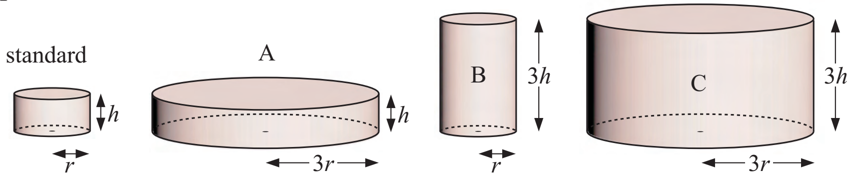
ii the total number of triangles in the first n rows

iii the total number of white triangles in the first n rows

iv the total number of black triangles in the first n rows.

b Show algebraically that your answer to **a ii** is the sum of your answers to **a iii** and **iv**.

9 Nov 2002, Paper 4



Sarah investigates cylindrical plant pots. The standard pot has base radius r cm and height h cm. Pot A has radius $3r$ and height h. Pot B has radius r and height $3h$. Pot C has radius $3r$ and height $3h$.

a **i** Write down the volumes of pots A, B and C in terms of π, r and h.

ii Find in its lowest terms the ratio of the volumes of A : B : C.

iii Which one of the pots A, B or C is mathematically similar to the standard pot? Explain your answer.

iv The surface area of the standard pot is S cm^2. Write down in terms of S the surface area of the similar pot.

b Sarah buys a cylindrical plant pot with radius 15 cm and height 20 cm. She wants to paint its outside surface (base and curved surface area).

i Calculate the area she wants to paint.

ii Sarah buys a tin of paint which will cover 30 m^2. How many plant pots of this size could be *completely* painted on their outside surfaces using this tin of paint?

10 Nov 2002, Paper 4

a Write down the 10th term and the nth term of the following sequences.

i 1, 2, 3, 4, 5, ..., ..., **ii** 7, 8, 9, 10, 11, ..., ..., **iii** 8, 10, 12, 14, 16, ..., ...

b Consider the sequence $1(8-7)$, $2(10-8)$, $3(12-9)$, $4(14-10)$, ..., ...

i Write down the next term and the 10th term of this sequence in the form $a(b-c)$ where a, b and c are integers.

ii Write down the nth term in the form $a(b-c)$ and then simplify your answer.

11 Nov 2000, Paper 4

A teacher asks four students to write down an expression using each of the integers 1, 2, 3 and n exactly once. Ahmed's expression was $(3n+1)^2$. Bumni's expression was $(2n+1)^3$. Cesar's expression was $(2n)^{3+1}$. Dan's expression was $(3+1)^{2n}$. The value of each expression has been worked out for $n=1$ and put in the table below.

a Copy and complete this table, giving the values for each student's expression for $n = 2, 0, -1$ and -2.

	$n=2$	$n=1$	$n=0$	$n=-1$	$n=-2$
Ahmed		16			
Bumni		27			
Cesar		16			
Dan		16			

b Whose expression will always give the greatest value **i** if $n < -2$ **ii** if $n > 2$?

c Cesar's expression $(2n)^{3+1}$ can be written as an^b and Dan's expression $(3+1)^{2n}$ can be written as c^n. Find the values of a, b and c.

d Find any expression, using 1, 2, 3 and n exactly once, which will always be greater than 1 for any value of n.

12 Adapted from Nov 1997, Paper 4

a A tin of soup is 11 centimetres high and has a diameter of 8 centimetres (Diagram 1). Calculate the volume of the tin.

Diagram 1

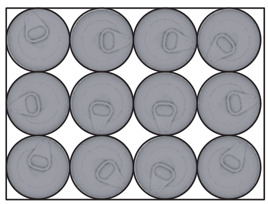


Diagram 2

b The tins are packed tightly in boxes of 12, seen from above in Diagram 2. The height of each box is 11 centimetres.

i Write down the length and the width of the box.

ii Calculate the percentage of the volume of the box which is not occupied by the tins.

c A shopkeeper sells the tins of soup for \$0.60 each. By doing this he makes a profit of 25% on the cost price. Calculate the cost price of

i one tin of soup **ii** a box of 12 tins.

d The shopkeeper tries to increase sales by offering a box of 12 tins for \$6.49. At this price:

i how much does a customer save by buying a box of 12 tins

ii what percentage profit does the shopkeeper make on each box of 12 tins?

13 Nov 1997, Paper 4

A "Pythagorean triple" is a set of three **whole** numbers that could be the lengths of the three sides of a right-angled triangle.

a Show that $\{5, 12, 13\}$ is a Pythagorean triple.

b Two of the numbers in a Pythagorean triple are 24 and 25. Find the third number.

c The largest number in a Pythagorean triple is x and one of the other numbers is $x-2$.

i If the third number is y, show that $y=\sqrt{4x-4}$.

ii If $x=50$, find the other two numbers in the triple.

iii If $x=101$, find the other two numbers in the triple.

iv Find two other Pythagorean triples in the form $\{y, x-2, x\}$, where $x<40$. Remember that all three numbers must be whole numbers.

14 Adapted from June 1991, Paper 4

a Show that

i $2\frac{1}{2}\times 1\frac{2}{3}=2\frac{1}{2}+1\frac{2}{3}$ **ii** $1\frac{3}{4}\times 2\frac{1}{3}=1\frac{3}{4}+2\frac{1}{3}$ **iii** $2\frac{1}{5}\times 1\frac{5}{6}=2\frac{1}{5}+1\frac{5}{6}$

b Write $2\frac{1}{2}$ and $1\frac{2}{3}$ in the form $1+\frac{x}{y}$ and repeat for $1\frac{3}{4}$ and $2\frac{1}{3}$.

c From your observations in **b**, find another statement like those in **a** which is true.

d Write down a generalisation of what you have discovered and prove it algebraically.

15 Adapted from June 1997, Paper 4

Maria thinks of 3 possible savings schemes for her baby son.

Scheme A: save \$10 on his 1st birthday, \$20 on his 2nd birthday, \$30 on his 3rd birthday, \$40 on his 4th birthday, ...

Scheme B: save \$1 on his 1st birthday, \$2 on his 2nd birthday, \$4 on his 3rd birthday, \$8 on his 4th birthday, ...

Scheme C: save \$1 on his 1st birthday, \$4 on his 2nd birthday, \$9 on his 3rd birthday, \$16 on his 4th birthday, ...

She puts these ideas in a table.

Scheme/Birthday	1st	2nd	3rd	4th
A	\$10	\$20	\$30	\$40
B	\$1	\$2	\$4	\$8
C	\$1	\$4	\$9	\$16

a Write down, for each of the Schemes A, B and C, the amount to be saved on

i his 7th birthday **ii** his nth birthday.

b The formulae for the *total* amount saved up to and including his nth birthday are as follows.

$$\text{Scheme A:}\quad \text{total} = \$5n(n+1)$$
$$\text{Scheme B:}\quad \text{total} = \$(2^n-1)$$
$$\text{Scheme C:}\quad \text{total} = \$\frac{n(n+1)(2n+1)}{6}$$

i For *each* of the schemes A, B and C, find the total amount saved up to and including his 10th birthday.

ii Which scheme gives the smallest total amount of savings up to and including his 18th birthday?

iii Find the birthday when the scheme you have selected in **b ii** first gives the smallest total amount of savings.

16 Adapted from June 1990, Paper 4

a Work out:

i 26×93 and 62×39 **ii** 36×42 and 63×24

b Find *one* other pair of multiplications with the same property.

c Explain why every two digit number can be written in the form $10a + b$ where $a, b \in \mathbb{Z}^+$.

d What can be deduced from the equation $(10m + n)(10r + s) = (10n + m)(10s + r)$?

17 Adapted from Nov 1992, Paper 4

n	1	2	3	4
n^2	1		9	
n^4	1		81	

a Copy and complete the table of values above.

b In the table below,

$$p = 1^2 + 2^2$$
$$q = 1^2 + 2^2 + 3^2 + 4^2$$
$$r = 3(2^2) + 3(2) - 1$$
$$s = 3(3^2) + 3(3) - 1$$
$$t = 1^4 + 2^4 + 3^4$$
$$u = 1^4 + 2^4 + 3^4 + 4^4$$

Calculate the values of p, q, r, s, t and u.

n	1	2	3	4	......	20
Row X	1	p	14	q	......	
Row Y	5	r	s	59	......	
Row Z	1	17	t	u	......	

c For the first four values of n in the table, consider the (Row X value) $\times$ (Row Y value) and the Row Z value. Find the formula which connects Row X and Row Y with Row Z.

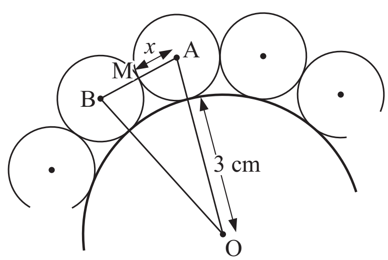
d **i** The value in Row X for $n = 20$ can be found by putting $n = 20$ into the formula $X = \dfrac{n(n+1)(2n+1)}{6}$. Find this value of X.

ii The value in Row Y for $n = 20$ can be found by putting $n = 20$ into the formula $Y = 3n^2 + 3n - 1$. Find this value of Y exactly.

e Use your answers to **c** and **d** to find the exact value of $1^4 + 2^4 + 3^4 + ... + 19^4 + 20^4$.

18 Adapted from Nov 1992, Paper 4

One central circle, of radius 3 cm and centre O, is completely surrounded by other circles which touch it and touch each other, as shown in the diagram. These outer circles are identical to each other.



a If the radius of each outer circle is x cm, write down the following lengths in terms of x:

i OA **ii** OB **iii** AB.

b On one occasion there are 6 circles completely surrounding the central circle.

i Calculate angle AOB **ii** What special type of triangle is AOB in this case?

iii Use your previous answers to find x.

c On another occasion there are 20 small circles completely surrounding the central circle.

i Calculate angle AOB.

ii M is the midpoint of AB. Consider the triangle MAO and write down the equation involving x and a trigonometric ratio.

iii Solve this equation to find x correct to 2 decimal places.

d Extend the result to n small circles and test your result when $n = 20$.

19 Adapted from Nov 1996, Paper 4

a As the product of its prime factors, $1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$.
Write 135, 210 and 1120 as the product of their prime factors.

b Copy this grid.

$a = 1$	$b =$	$c =$
$d =$	$e =$	$f =$
$g =$	$h =$	$i = 8$

The nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are to be placed in your grid in such a way that the following four statements are *all* true.

$$a \times b \times d \times e = 135$$
$$b \times c \times e \times f = 1080$$
$$d \times e \times g \times h = 210$$
$$e \times f \times h \times i = 1120$$

The digits 1 and 8 have already been placed for you.
Use your answers to **a** to answer the following questions.

i Which is the only digit, other than 1, that is a factor of 135, 1080, 210 and 1120?

ii Which is the only letter to appear in all four statements above?

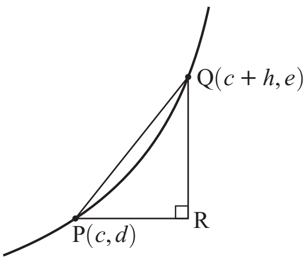
iii 7 is a factor of only two of the numbers 135, 1080, 210 and 1120. Which two?

c Now complete your grid.

20 June 1994, Paper 4

a Calculate the gradient of the straight line joining the points (3, 18) and (3.5, 24.5).

b The diagram shows part of the curve $y = 2x^2$.



i P is the point (c, d). Write down d in terms of c.

ii Q is the point $(c + h, e)$. Write down e in terms of c and h.

iii Write down the length of PR. Find an expression for the length of QR in terms of c and h, and simplify your answer.

iv Show that the gradient of the line PQ is $4c + 2h$.

v If P is the point (3, 18) and Q is the point (3.5, 24.5), state the value of c and the value of h, and use these values to show that **b iv** gives the same answer as **a**.

vi If P is the point (3, 18) and Q is the point (3.1, 19.22), state the value of c and the value of h, and use **b iv** to find the gradient of the line PQ.

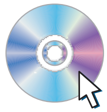
vii If P is the point (3, 18) and Q gets closer and closer to P, what happens

a to the value of h

b to the value of the gradient of the line PQ?

Click on the icon to obtain 4 more investigation questions.

PRINTABLE QUESTIONS



B MODELLING QUESTIONS

1 June 1990, Paper 4

A gardener has 357 tulip bulbs to plant.

a If she planted a rectangle of 15 rows, with 23 bulbs in each row, how many bulbs would be left over?

b How many bulbs would there be in the largest square that she could plant?

c **i** If she plants x rows, with y bulbs in each row, write down a formula for the number of bulbs left over.

ii If $10 < x < 20$ and $y > 20$, find the value of x and the value of y such that no bulbs are left over.

2 The depth of water (d metres) in a harbour is given by the formula $d = a + b\sin(ct)^o$ where a, b and c are constants, and t is the time in hours after midnight. It is known that both b and c are non-zero and $20 < c < 35$.

The following table gives depths at particular times:

t	midnight	noon	1300	1400
d	5	5	7	8.46

a Using the first three pieces of information in the table, deduce the values of a, b and c.

b Check that the formula is correct by substituting the fourth piece of information.

c Find the depth of water at 10 am.

d What is the greatest depth of water in the harbour?

e At what times of day is the depth of water greatest?

f What is the least depth of water in the harbour?

3 Adapted from Nov 1994, Paper 4

a In a chemical reaction, the mass M grams of a chemical is given by the formula $M = 160a^{-t}$ where a is a constant integer and t is the time (in minutes) after the start. A table of values for t and M is given below.

t	0	1	2	3	4	5	6	7
M	160	80	40	20	q	5	r	1.25

i Find the value of a.

ii Find the values of q and r.

iii Sketch the graph of M against t.

iv Draw an accurate graph and add to it a tangent at $t = 2$. Estimate the rate of change in the mass after 2 minutes.

b The other chemical in the same reaction has mass m grams where $m = 160 - M$.

i On the same graph as in **a iii**, sketch the graph of m against t.

ii For what value of t do the chemicals have equal mass?

iii State a single transformation which would give the graph for m from the graph for M.

4 The surge model has form $y = \dfrac{at}{2^{bt}}$ where a and b are constants and t is the time, $t \geqslant 0$.

This model has extensive use in the study of medical doses where there is an initial rapid increase to a maximum and then a slow decay to zero.

a Use a graphics calculator to graph the model (on the same set of axes) for:

i $a = 10,\ b = 2$ **ii** $a = 15,\ b = 3$

b The effect of a pain killing injection t hours after it has been given is shown in the following table:

Time (t hours)	0	2	4	6	12
Effect (E units)	0	25	25	r	s

The effect E follows a surge model of the form $E = \dfrac{at}{2^{bt}}$.

i By using *two* of the points of this table, find the values of a and b.

ii Hence, find the values of r and s in the table.

iii Use your calculator to find the maximum effect of the injection and when it occurs.

iv It is known that surgical operations can only take place when the effectiveness is more than 15 units. Between what two times can an operation take place?

5 The logistic model has form $y = \dfrac{a \times 2^{bt}}{2^{bt} + c}$ where t is the time, $t \geqslant 0$. The logistic model is useful in describing limited growth problems, i.e., when the y variable cannot grow beyond a particular value for some reason.

a Use technology to help graph the logistic model for $a = 3$, $b = \frac{1}{2}$ and $c = 2$. (Use the window $-1 \leqslant x \leqslant 40$, $-1 \leqslant y \leqslant 5$.)

b What feature of the graph indicates a limiting value?

c What is the limiting value?

d Bacteria is present in a carton of milk and after t hours the bacteria (B units) was recorded as follows:

t	0	1	2
$B(t)$	10	12.70	15.03

It is known that $c = 1$.

i Use the first two sets of data to find a and b, and hence determine the logistic model.

ii Use the model found in **i** to check the third data set.

iii What is the limiting quantity of bacteria for this model?

iv In the general model $y = \dfrac{a \times 2^{bt}}{2^{bt} + c}$, explain why the limiting quantity has value a.

6 **June 1991, Paper 4**

A farmer keeps x goats and y cows. Each goat costs \$2 a day to feed and each cow costs \$4 a day to feed. The farmer can only afford to spend \$32 a day on animal food.

a Show that $x + 2y \leqslant 16$.

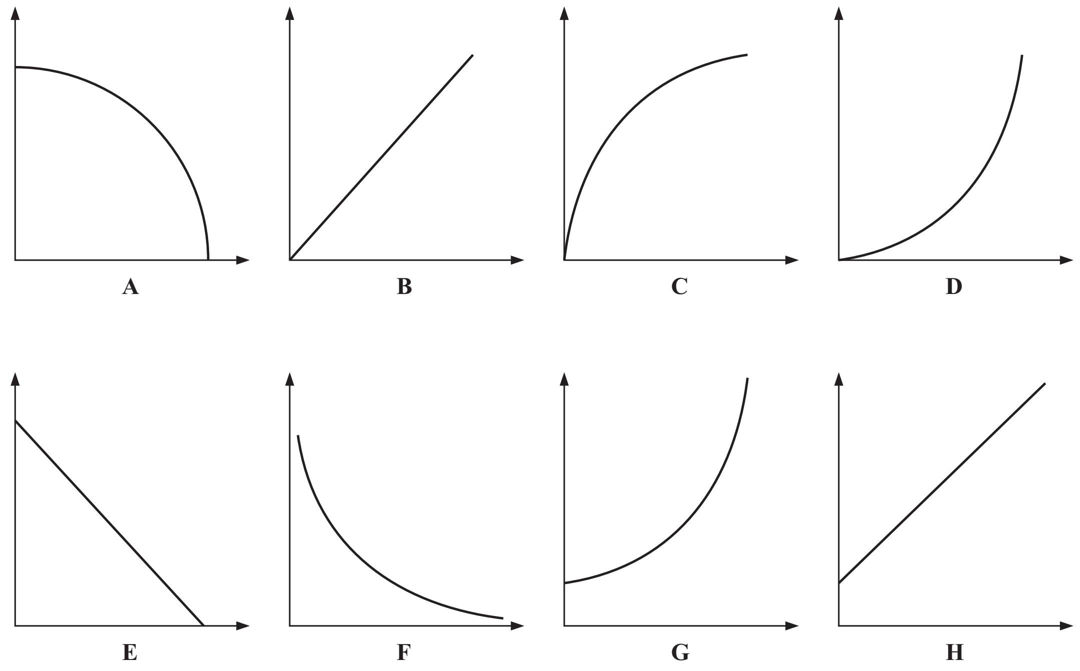
b The farmer has room for no more than 12 animals. He wants to keep at least 6 goats and at least 3 cows. Write down three more inequalities.

c Using a scale of 1 cm to represent 1 unit on each axis, represent the four inequalities on a graph.

d One possible combination which satisfies all the inequalities is 6 goats and 4 cows. Write down all the other possible combinations.

e If he makes a profit of \$50 on each goat and \$80 on each cow, which combination will give him the greatest profit? Calculate the profit in this case.

7 May 2001, Paper 4



a Write down which one of the sketch graphs above labelled **A** to **H** shows each of the following:

i a *speed-time* graph for a car which starts from the rest and has constant acceleration

ii $y = x^3 + 1$

iii y is inversely proportional to x^2

iv the sum of x and y is constant

v $y = \cos x$ for $0^o \leqslant x \leqslant 90^o$

vi a *distance-time* graph when the speed is decreasing.

b Write down an equation for sketch graph **D** if it passes through the points (1, 1) and (2, 4) and, when extended to the left, has line symmetry about the vertical axis.

8 June 1994, Paper 4

In a school gardening project, teachers and students carry earth to a vegetable plot. A teacher can carry 24 kg and a student can carry 20 kg. Each person makes one trip. Altogether at least 240 kg of earth must be carried. There are x teachers and y students.

a Show that $6x + 5y \geqslant 60$.

b There must not be more than 13 people carrying earth, and there must be at least 4 teachers and at least 3 students. Write down three more inequalities in x and/or y.

c **i** Draw x and y axes from 0 to 14, using 1 cm to represent 1 unit of x and y.

ii On your grid, represent the information in parts **a** and **b**. Shade the *unwanted* regions.

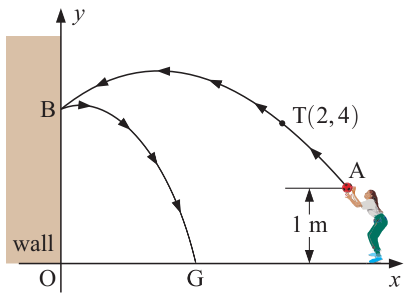
d From your graph, find

i the least number of people required

ii the greatest amount of earth which can be carried.

9 **Adapted from Nov 1993, Paper 4**

Anna throws a ball from a point A, one metre above the ground, towards a wall. The ball travels along the arrowed path from A to B, given by the equation $y = a + bx - x^2$ where the x-axis represents the horizontal ground and the y-axis represents the wall. The ball passes through the point T(2, 4) and hits the wall 4 m above O.



a Find the values of a and b.

b Show that the x-coordinate of A satisfies the equation $x^2 - 2x - 3 = 0$.

c Find the x-coordinate of A.

d The ball rebounds from the wall at B to the ground at G. The equation of the path B to G is $y = c - 2x - x^2$.

i Find the value of c. **ii** Find the x-coordinate of G correct to 2 decimal places.

e How far from the wall is the ball when it is **i** 0.5 m **ii** 3 m above the ground?

f Find the greatest height of the ball during its motion.

10 **May 2005, Paper 4**

A taxi company has "SUPER" taxis and "MINI" taxis. One morning a group of 45 people needs taxis. For this group the taxi company uses x "SUPER" taxis and y "MINI" taxis. A "SUPER" taxi can carry 5 passengers and a "MINI" taxi can carry 3 passengers. So $5x + 3y \geqslant 45$.

a The taxi company has 12 taxis. Write down *another* inequality in x and y to show this information.

b The taxi company always uses at least 4 "MINI" taxis. Write down an inequality in y to show this.

c Draw x and y axes from 0 to 15 using 1 cm to represent 1 unit on each axis.

d Draw three lines on your graph to show the inequality $5x + 3y \geqslant 45$ *and* the inequalities from parts **a** and **b**. Shade the *unwanted* regions.

e The cost to the taxi company of using a "SUPER" taxi is \$20 and the cost of using a "MINI" taxi is \$10. The taxi company wants to find the cheapest way of providing "SUPER" and "MINI" taxis for this group of people. Find the *two* ways in which this can be done.

f The taxi company decides to use 11 taxis for this group.

i The taxi company charges \$30 for the use of each "SUPER" taxi and \$16 for the use of each "MINI" taxi. Find the two possible *total* charges.

ii Find the largest possible *profit* the company can make, using 11 taxis.